Final Report for the Literacy and Numeracy Secretariat:

Niagara Catholic District School Board's Early Learning Math Inquiry Project (2013-2014)

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EXECUTIVE SUMMARY

Background

The program of professional learning that was the focus of this research sought to enhance early childhood educators' practices in mathematics instruction. Problem solving is the main context for mathematics learning and math talk is the vehicle for sharing and developing thinking. Within a math-talk community students engage in questioning, explain their mathematical thinking, describe sources of mathematical ideas, and assume responsibility for their own learning. Engaging with the big ideas in number sense and numeration allows students to explore the concepts in depth and see the connections to other concepts. This may be accomplished through inquiry-based instructional practices that engage students in co-operative, hands-on activities that make connections and applications between mathematics and other contexts. Teachers' use of inquiry-based instruction is influenced by their beliefs in its effectiveness. There is a need to support teachers' professional learning with respect to the role of language in mathematics inquiry-based teaching, while attending to their beliefs and attitudes about mathematics instruction and their content knowledge. A mathematics facilitator should support teachers as they explore pedagogies, mathematics concepts, and work in collaborative groups. Reflection is also an important consideration in teacher professional learning in mathematics. These components were key to the professional learning evaluated here which sought to build on early learning educators' mathematics content and pedagogical knowledge while bearing in mind their beliefs and attitudes about mathematics instruction and how students learn mathematics.

Methodology

The teacher professional learning in these schools focused on supporting students' conceptual understanding of number sense and numeration (e.g., cardinality, identifying symbols, composing/decomposing numbers, place and value) while participating in a collaborative, inquiry-based professional learning community. A Numeracy Facilitator led all professional learning sessions and worked with the Numeracy Consultant and two Math Coaches to offer ongoing support for the teachers throughout the project. There were two plenary sessions that were attended by all teacher participants and five, half day sessions that were guided by the Numeracy Facilitator at each of the school sites.

This research was an evaluative case study with the purpose to inquire into an educational program in order to determine its effectiveness. There were five research questions that related to educators' practices, beliefs, attitudes and students' achievement. Quantitative and qualitative data (surveys, interviews, fieldnotes, learning blogs, assessment scores) were collected from four sets of participants: teachers, early childhood educators, facilitators, and students. There were 16 teachers (ELKP to Grade 2) and 8 Early Childhood Educators (ECE) at three schools that participated in the program and their students (n=245) indirectly participated as recipients of the educators' professional learning. All participants signed informed consent forms to volunteer for the data collection.

Findings

The use of a diagnostic or assessment for learning measure provided teachers and early childhood educators with achievement data in relation to specific numeracy concepts to make instructional decisions accordingly. This moved the educators' orientation away from a deficiency perspective focussed on what the student *does not* know and prioritized student learning around foundational instructional concepts such as the eight principles of counting.

To foster dynamic inquiry-based instruction, educators allowed students to take the lead in their own learning, using students' interests as a catalyst. This supported students' knowledge of multiple ways to solve problems, make meaningful schematic connections, and it kept students engaged in the learning process. As well, teaching through problem solving with manipulatives and visuals was successful at supporting students' conceptual understandings, fostering student engagement and

making abstract concepts more meaningful and relevant. Educators intend to continue to teach number sense using inquiry and multiple strategies to accommodate for students' individual differences.

The importance of focusing on students' learning process, not just the product of their work, was identified as a way to promote students' understanding of concepts. Students were required to explain their thinking and talk about strategies to reinforce the importance of processes and appeal to higher-order thinking. Interactive dialogue with the student was highlighted as an important vehicle for cultivating an accurate picture of students' understanding. Educators became more aware of how they wanted to continue to use language in relation to how they talked about numbers. Math talk needs to be modelled and embedded into the culture of the classroom.

Educators investigated the progression of number sense expectations from primary grades into the junior division and recognized that number sense is pervasive throughout the curriculum. As a result, number sense could be effectively taught by integrating curriculum expectations into lessons rooted in other curricular strands and this appeared to be an efficient way to cover curriculum.

While there appears clear support for implementing evidence-based practices in the classroom, it can be difficult for teachers to know the extent to which they should be using such practices. Despite the consistent pressure of time constraints on classroom activities, teachers began covering material more thoroughly because they gave themselves permission to trust their instructional instincts enough to slow down and cover material comprehensively.

Shifts in educators' beliefs about math instruction occurred. Educators are less likely to hold the belief that mathematics problems are solved by proposing an absolute solution – there are multiple pathways to process mathematics questions. Similarly, these educators are more likely to believe that mathematics is a dynamic of many different ideas and learners interpret and organize this dynamic of information. These educators do not contend that mathematics learning is demonstrated through computations and the ability to memorize facts, procedures or formulae. In a similar vein, these educators do not believe that students should be focused on quickly getting a correct right answer to a mathematics problem – the process of problem solving and understanding why and how one derived at a solution is of great value. The educators regard a mathematics learning context as one that is enhanced by challenging mathematics problems within a supportive environment, and activities which build upon and respect students' experiences. The educators now appreciate their key role in monitoring and being responsive to students' activities in mathematics.

Educators welcomed the opportunity for growth, as they recognized they were at a critical inflection point on their own learning curve and that in order to promote student achievement they needed to invest in building their personal capacity. Accordingly, educators perceived their new instructional strategies and practices discussed at facilitated sessions to be of value to their instructional repertoire. Educators appreciated having access to support, both human support in their classroom to model best practices, and access to tangible resources. Support was a positive influence in their efforts around instructional capacity.

The results of the analyses of students' conceptual understanding and achievement were noted in students' transfer of skills to new settings and a demonstration of their own initiative without prompting. Students appear to be more confident in math, more comfortable with open ended tasks, and willing to take risks. Quantitative results derived from a pre-test and post-test of students' mathematical concepts complement these qualitative findings. There was a significant growth in student performance for all students (ELKP, Grade 1, 2) at all three schools in the majority of assessed concepts.

Implications

Based on the findings, implications for practice and future research are offered. Facilitators should begin with a discussion of how to foster a student-driven classroom culture that supports inquiry-based strategies. On-site instructional modeling and demonstrations of new strategies might set the stage for educators to translate into their own practice. An expansion of the current program of professional learning might include a blend of teachers that have previously participated and teachers new to the initiative. Comprehensive baseline data about students' knowledge in relation to curricular expectations needs to be collected early in the school year.

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Background and Identified Need

Mathematics Achievement in the Province: Niagara Catholic District School Board's Response

The mathematics achievement of students in the Province of Ontario has consistently dropped from Grade 3 to 6 over the past five years. **Figures 1.** and **2**. display the percentage of all students (in Grades 3 and 6 respectively) at or above the provincial standard. **Figure 3.** shows the longitudinal changes in students' achievement from Grade 3 to 6.

Percentage of All Grade 3 Students at or Above the Provincial Standard (Levels 3 and 4) Over Time

	2008–2009	2009–2010	2010–2011	2011–2012	2012–2013
NUMBER OF STUDENTS	# = 125 481	# = 127 789	# = 124 117	# = 126 455	# = 127 645
READING	61%	62%	65%	66%	68%
WRITING	68%	70%	73%	76%	77%
MATHEMATICS	70%	71%	69%	68%	67%

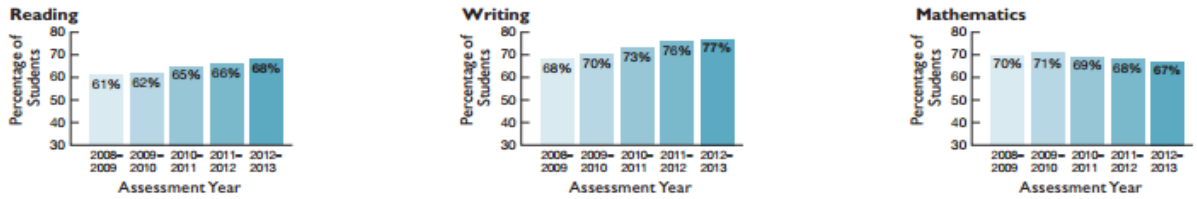

Figure 1. Grade 3 Students at or Above Provincial Standard

Percentage of All Grade 6 Students at or Above the Provincial Standard (Levels 3 and 4) Over Time

	2008–2009	2009–2010	2010–2011	2011–2012	2012–2013
NUMBER OF STUDENTS	# = 136 076	# = 134 294	# = 132 308	# = 129 477	# = 131 589
READING	69%	72%	74%	75%	77%
WRITING	67%	70%	73%	74%	76%
MATHEMATICS	63%	61%	58%	58%	57%

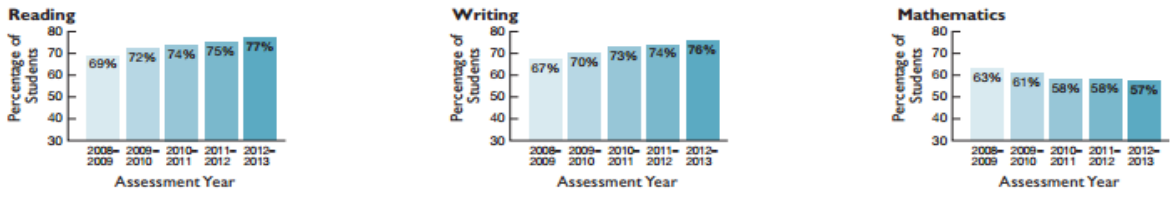

Figure 2. Grade 6 Students at or Above Provincial Standard

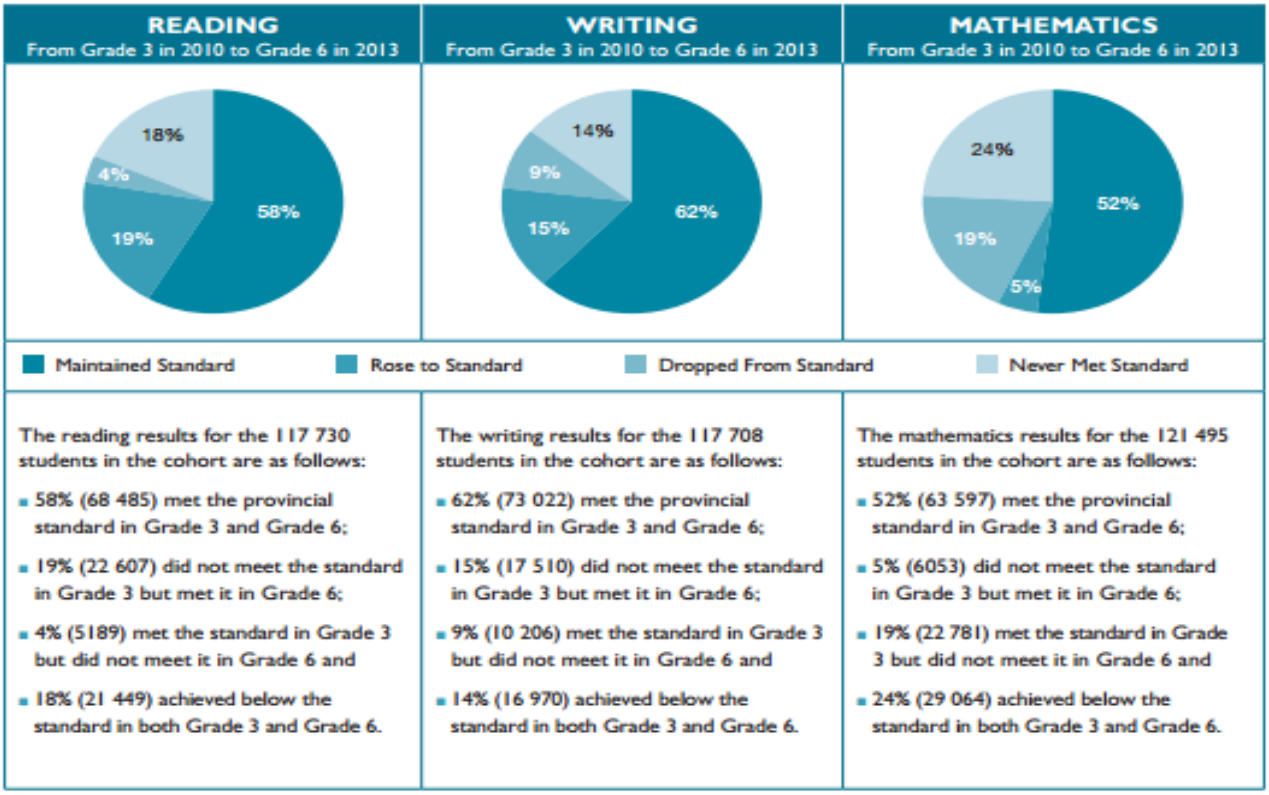

Figure 3. Change in Students' Achievement from Grades 3 to 6

In addition to this Provincial achievement data, the accomplishments of the previous two-year, *Junior Mathematics Intervention Project* provided the rationale for the Niagara Catholic District School Board to embark on a journey to solidify the number sense of students in ELKP-2. If there were significant gaps in student achievement with respect to number sense and these gaps could be closed with targeted intervention, then the instructional focus needed to be in the early years. The inquiry question that NCDSB adopted was:

> *If we provide highly effective mathematics instruction focusing on Number Sense and Numeration in ELKP – Grade 2, then our students will have a deeper understanding of the concept of number and be able to use that understanding in a flexible manner to solve problems.*

The Niagara Catholic District School Board referenced the work of Duncan (2011) which identified six population-based data sets involving 16,387 children and included measures of reading and math competency, attention skills, pro-social behavior, and antisocial and internalizing behavior. Data were collected at the time of school entry, and then measures of reading and math competency were taken later in the primary or middle school years. This analysis is widely viewed as providing a clear answer about the relative role of school-entry skills and behaviors: early academic skills appear to be the strongest predictor of subsequent scholastic success – early math skills more so than early reading skills. Duncan asserts that research should now focus on why math skills, which combine conceptual and procedural competencies, are the most powerful predictor of subsequent achievement and attainment. Duncan (2011, as cited in Christensen, 2001), states that "experimental evaluations of early math programs that focus on particular skills and track children's reading and math performance throughout elementary school could help identify missing causal links between early skills and later success."

Theoretical and Practical Foundations

Theoretical Foundations: Academic Research

For over a decade, there has been an emphasis on supporting early learners as they engage with the big ideas in mathematics. There are five representations of mathematical ideas: pictures, written symbols, manipulative models, real-work situations, and oral language (Van De Walle, Karp, & Bay-Williams, 2013). Translations between and within each of these five representations can support understanding and aid learning new concepts. To foster an understanding of such broad new concepts, teachers should teach using these five representations. This call to action is premised on the notion of supporting "mathematical literacy which includes the roles of language, symbols, texts and social interactions in the early development of mathematical thinking and understanding" (Ashton, 2007). This notion implies that context and communication are integral to the application of mathematical representations.

Problem solving is the main context for mathematics learning and math talk is the vehicle for sharing and developing thinking (Ministry of Education, 2003). Engaging with the big ideas in number sense and numeration allows students to explore the concepts in depth and see the connections to other concepts. Using the big ideas in mathematics helps teachers to see that concepts should not be taught in isolation, but as "a connected network of interrelated concepts" (Ministry of Education, 2003). By teaching the big ideas in number sense, teachers are appealing to a depth of understanding as these big ideas are conceptually interdependent and overlapping. Implicit is the need to teach for this profundity in conceptual understanding, knowledge and language and not to cover a breadth of mathematical skills.

The program of professional learning that was the focus of this research sought to enhance early childhood educators' practices in mathematics instruction. Recommendations set out by the National Association for the Education of Young Children (NAEYC) and the National Council of Teachers of Mathematics (NCTM; 2010) were inherent in the mathematics programming espoused by the Niagara Catholic District School Board (NCDSB). These state that teachers of mathematics for 3- to 6-year-old children should:

1. enhance children's natural interest in mathematics and their disposition to use it to make sense of their physical and social worlds
2. build on children's experience and knowledge, including their family, linguistic, cultural, and community backgrounds; their individual approaches to learning; and their informal knowledge
3. base mathematics curriculum and teaching practices on knowledge of young children's cognitive, linguistic, physical, and social-emotional development
4. use curriculum and teaching practices that strengthen children's problem-solving and reasoning processes as well as representing, communicating, and connecting mathematical ideas
5. ensure that the curriculum is coherent and compatible with known relationships and sequences of important mathematical ideas
6. provide for children's deep and sustained interaction with key mathematical ideas
7. integrate mathematics with other activities and other activities with mathematics
8. provide ample time, materials, and teacher support for children to engage in play, a context in which they explore and manipulate mathematical ideas with keen interest
9. actively introduce mathematical concepts, methods, and language through a range of appropriate experiences and teaching strategies
10. support children's learning by thoughtfully and continually assessing all children's mathematical knowledge, skills, and strategies.

When teachers believe that they command a strong understanding of mathematics concepts, their students believe that they can generate ideas and solutions to mathematics problems and that math knowledge does not originate from external sources (Beghetto & Baxter, 2012). Yet, teachers hold different beliefs about the autonomy of students to construct mathematics knowledge through language

and their own autonomy to make instructional decisions. This reality was an important consideration for the program of professional learning featured here.

In general, teachers hold distinct beliefs about how their instruction is affected by their role, their students and the content (Elmore, 2009). Traditional, teacher-centred instruction contends that the role is teacher-as-director and students are recipients of the content. Contemporary, inquiry-based instructional practices engage students in hands-on activities with manipulatives to explore concepts and work on problems for which there is no immediately obvious method of solution (Wilkins, 2008). Nonetheless, not all teachers believe that students can self-direct their own mathematics learning. In a study of elementary teachers who focused their professional learning on how to enhance students' problem-solving strategies, only about half of the teachers involved the students in inquiry into their peers' strategies (Warfield, Wood & Lehman, 2005). By contrast, teachers who believe that inquiry-based instructional practices are effective will engage students in co-operative, hands-on activities that make connections and applications between mathematics and other contexts (Wilkins, 2008).

Teachers are beginning to value the utility of language and talk as integral to the dynamic of teaching mathematics through inquiry (Hye & Reifel, 2011). Students need to be engaged in communication with their peers while their teacher poses questions and allows them to explore potential solutions to mathematics problems. Teachers can elicit math-talk that actively engages students as they interact and explain their thinking. A math-talk learning community is defined as one in which students assist each other by engaging in meaningful mathematical discourse. Within this community students engage in questioning, explain their mathematical thinking, describe sources of mathematical ideas, and assume responsibility for their own learning (Hufferd-Ackles, Fuson, & Sherin, 2004). Teachers can effectively support primary level students to engage in math-talk that includes interrogation, explanations of their thinking processes and argument (Bruce & Flynn, 2011). Research has found that primary teachers can effectively support students in their use of talk as a tool for mathematical reasoning and problem solving (Mercer & Sams, 2006).

In a math-talk community, teachers should pose open-ended or open-routed questions. To do so effectively, teachers need to have experience in devising pedagogically meaningful questions and supporting students' responses that exemplify their efforts to make meaning and articulate their thinking. Teachers need to give pedagogically sound explanations that bridge mathematical content to students' thinking (Inoue & Buczynski, 2011). When teachers become active participants listening and talking during student conversations, they are able to help students express their thinking, assess students' understanding, and determine misconceptions (Vanderhye & Demers, 2008).

Accordingly, it is essential to address student misconceptions in mathematics through assessment to diagnose student needs (Airasian, Engemann, & Gallagher, 2010; Ministry of Education, 2010) and inform instructional decision-making. Good assessment practices provide information on student conceptual and skill development as well as information on the effectiveness of math-talk, questioning, and instructional activities. Teachers need to constantly assess and evaluate their teaching strategies to determine if they are supporting inquiry in the classroom (Jacobs, 2004) and facilitating the learning of big ideas in mathematics (Ministry of Education, 2006). In the early learning classroom, teachers should observe students to assess how they demonstrate an understanding of a concept, use strategies to solve the problem and then explain the strategies that they used.

The academic research points to the need to support teachers' professional learning with respect to the role of language in mathematics problem-solving teaching, while attending to their beliefs and attitudes about mathematics instruction and their content knowledge. Since beliefs, attitudes, and content knowledge are all related to teachers' mathematics instructional practice (Wilkins, 2008), a program of professional learning should address all of these components. In particular, attention must be paid to teachers' beliefs as these mediate the effects of content knowledge and instructional practice attitudes (Wilkins, 2008). Specifically, how do these factors interact with respect to inquiry-based mathematics instruction?

Teachers' use of inquiry-based instruction is influenced by their beliefs in its effectiveness. Not surprisingly, teachers with positive attitudes toward mathematics are more likely to believe in the effectiveness of inquiry-based instruction and use it more frequently in their classroom. Interestingly, a

recent study by Wilkins (2008) found that early primary teachers (grades K-2) tend to naturally use inquiry-based instructional methods more often than junior teachers (grades 3-5).

The design of a professional learning program in mathematics is essential. In particular, when the program of professional learning focuses on how to incorporate and pose effective mathematics problems, teachers' beliefs shift toward recognizing the impact of their mathematics instruction (Barlow & Cates, 2006). A successful program of professional learning for elementary teachers found that modelling self-questioning during mathematical problem solving combined with systematic reflective support from colleagues contributes to teachers' pedagogical-content knowledge and their metacognitive knowledge of mathematics (Kramarski, 2009). The program of professional learning evaluated here sought to build on early learning educators' mathematics content and pedagogical knowledge while bearing in mind their beliefs and attitudes about mathematics instruction and how students learn mathematics.

Supporting the practice and confidence of teachers to instruct mathematics through problem solving takes time and personnel (e.g., facilitators, consultants, coaches). Research (Obara & Sloan, 2009) on the role of a mathematics facilitator has found that teachers benefit from a site-based, long-term professional learning that targets content and pedagogical knowledge. The mathematics facilitator should support teachers as they explore pedagogies, mathematics concepts, and work in collaborative groups. Teachers also need opportunities to personally engage and practice problem solving together. This type of collaboration contributes to teachers becoming more comfortable with the mathematics and recognizing the importance of group work while problem solving. For most teachers, this transfers to how they then support their students' collaborative problem solving (Sakshaug & Wohlhuter, 2010).

Reflection is also an important component in teacher professional learning in mathematics. It has been found that teachers' individual reflection facilitates the reflexive relationships within a school's community of practice. Reflection enables teachers to improve their skills, beliefs and perceptions of mathematics teaching and is an overall facilitator of teacher development (Turner, 2009). The *Early Learning Math Inquiry Project* in NCDSB was conceived with these professional learning design components in mind.

Practical Foundations: Ministry of Education and Literacy and Numeracy Secretariat Resources

Paying Attention to Mathematics Education K-12 (Ministry of Education, 2011)

The Ministry of Education (2011) cites two factors that contribute to successful projects focused on mathematics instruction and student achievement. These two factors were integrated into *NCDSB's Early Learning Math Inquiry Project*:

1. collaboration and learning that includes ***teacher professional development meetings and classroom*** **experiences** for students and teachers;
2. use of mathematics content and pedagogical expertise that can ***support the embedding of understanding of mathematics*** content for teaching and learning within the ongoing work of teachers.

School Effectiveness Framework (2010)

Key components of NCDSB's *Early Learning Math Inquiry Project* are the effective practices of *assessment for, as, and of learning* as well as *School and Classroom Leadership*. The following indicators from the *School Effectiveness Framework (2010)* were instrumental factors within this Project:

Indicator 1.5:
A variety of valid and reliable assessment data is used by students and teachers to continuously monitor learning, to inform instruction and assessment to determine next steps.

Specifically:
- Instructional decisions are made and actions are taken to respond to what student demonstrations reveal

Indicator 2.4:
Job-embedded and inquiry-based professional learning builds capacity, informs instructional practice and contributes to a culture of learning.
Specifically:
- Evidence of student learning is shared as a catalyst for professional dialogue.
- Knowledge and effective instructional practices are shared (through observation, co-planning, collaboration, mentoring and coaching)

Practical Foundations: Professional Resources

Instructional Rounds in Education *(Elmore, 2009)*

The *Early Learning Math Inquiry Project* cited Elmore's (2009) description of Cohen and Ball's instructional core: the relationship between the teacher, the student and the content. Based on this instructional core, Elmore's first principle challenges facilitators to:

1) increase the level of knowledge and skill that the teacher brings to the instructional process;
2) increase the level of complexity of the content the students are asked to learn;
3) change the role of the student in the instructional process.

In addition to these challenges, Elmore also outlines his considerations for effective professional development, noting its quality depends on:

1) what teachers are being asked to learn;
2) how they are learning it;
3) whether they can make the practices they are being asked to try, work in their classrooms.

Throughout the planning process, the *Early Learning Math Inquiry Project* facilitators frequently cross-referenced these six guidelines with the goals that were set for the professional learning sessions to ensure the cohesiveness of the project.

Teaching Student-Centred Mathematics *(Van de Walle & Lovin, 2006)*

Van de Walle and Lovin's *Teaching Student-Centred Mathematics K-3* provides NCDSB schools with a source for further content-knowledge-building and ideas for group-learning tasks. This resource allows teachers to locate relevant information and activities to help plan for targeting student misconceptions. All teacher participants were provided with an additional school set of this professional resource.

Research Questions

Research questions that framed this case study were posed to evaluate the Niagara Catholic District School Board's *Early Learning Math Inquiry Project* (2013-2014). The five research questions that were identified relate to teachers' (both classroom teachers and early childhood educators) practices and beliefs, and students' conceptual understand and achievement. The following are these research questions and sub-questions that guided the evaluation of this initiative:

Teachers' Practices

1. Are teachers (ELKP, Grades 1-2) using evidence-based practices in their mathematics Number Sense and Numeration instruction?
    a. Are teachers focusing on the relationship between the teacher, the student and the concepts?
    b. Are teachers identifying curriculum connections and using curriculum mapping?
    c. Are teachers promoting students' understanding of math content?
    d. Are teachers using instructional strategies such as: mathematical discourse, collaborative inquiry, teaching through problem solving, open questions and parallel tasks?

Teachers' Beliefs and Attitudes

2. Do teachers perceive growth in their knowledge of mathematics content and mathematics instructional methods in Number Sense and Numeration?
3. Has the recent project in professional learning in mathematics instruction increased teachers' self-efficacy in mathematics instruction?
4. Has the recent project in professional learning in mathematics instruction made an impact on teachers' intentions for their future practice?

Students' Conceptual Understanding and Achievement

5. Were there shifts in students' (ELKP - Grade 2) conceptual understandings and mathematical achievement in Number Sense as a function of their teachers' participation in the (*Early Learning Mathematics Inquiry*) project?

Research Design

Intervention Design

The intervention design has overlapping layers of teacher professional learning and enhanced student learning in mathematics. Three schools (of a similar demographic profile) participated for the first time and focused on implementing and supporting successful instructional practices in order to focus on student mathematics achievement in the Early Learning Years (ELKP, Grades 1-2). The targeted strand in mathematics was Number Sense and Numeration with the view of ensuring that students have a firm foundation prior to the end of the primary grades. The teacher professional learning in these schools focused on supporting students' conceptual understanding of number sense and numeration (e.g., cardinality, identifying symbols, composing/decomposing numbers, place and value) while participating in a collaborative inquiry based professional learning community. Both classroom teachers and early childhood educators (in the ELKP classrooms) participated. In order to address this intention, these schools were introduced to the document, *Paying Attention to Mathematics*

Education (Ministry of Education, 2011) and the "Seven Foundational Principles for Improvement in Mathematics K-12."

Operationally, the intervention design included three key components: facilitated teacher professional learning sessions, collegial teacher professional learning, and targeted student instruction (based on diagnostic assessment) The project focused on providing teachers with professional learning and support in the use of the instructional methods of student inquiry. This was accomplished through professional learning sessions that sought to clarify mathematics content and enhance teachers' pedagogical expertise. It was also the intention of this initiative to increase teachers' efficacy by building their self-confidence in their abilities to develop accurate conceptions to remedy the misconceptions that commonly occur with early learners' understanding of mathematics. Taken together, this project sought to improve student conceptual understandings and create a classroom culture of student inquiry in mathematics.

Facilitated Teacher Professional Learning Sessions

The teacher professional learning was facilitated in three NCDSB elementary schools. Teachers and early childhood educators in ELKP, Grades 1 and 2 participated in the project from October 2013 – May 2014. The Numeracy Facilitator (Carlo), Numeracy Consultant (Bettina) and two Numeracy Coaches (Luciana, Jessa) provided the teacher professional learning sessions to support:

- teaching through inquiry (e.g. teachers finding the mathematics in the work that they students were doing)
- strategies to target misconceptions identified through a diagnostic resources (e.g., *Developmental Math Assessment*)
- improve teacher efficacy through development of mathematics content and pedagogical knowledge

The Numeracy Facilitator led all professional learning sessions and offered ongoing support for the individual needs of teachers throughout the project. The schedule of facilitated professional learning sessions included one half plenary day, followed by five half days (approximately one per month), and then finished with one half day plenary. All participating teachers and early childhood educators had release time for these sessions.

The two plenary sessions were attended by all teacher participants and were co-facilitated by the Numeracy Consultant and two Numeracy Coaches. The first session in October, 2013 was an introductory session and the agenda included the following:

- overview of the research project
- key constructs: collaboration, inquiry, and number sense and numeration
- project vision and goals
- student assessment data
- progression from diagnostic assessments to responsive lesson planning to implementing inquiry in the classroom
- connections to mathematics expectations

The five, half day sessions were facilitated by the Numeracy Facilitator at each of the school sites and included in separate groupings the ELKP teaching team (teachers and early childhood educators) and the Grade 1 and 2 teachers. School administrators floated in and out of these sessions. These five sessions followed an identical agenda at each site which included guided activities for enhancing teachers' mathematics content and pedagogical knowledge and then group discussion. This discussion varied from session to session, however, the focus was typically on the school-site inquiry question, analysis of the mathematics diagnostic test results to identify student misconceptions; collaborative inquiry methods; teaching through student inquiry. There was time devoted during each of these

sessions for teacher participants to share their experiences and successes with their colleagues and write in their on-line learning logs.

The final plenary session in May, 2014, was a consolidation and debriefing session for all teachers and early childhood educators. The discussion focused on:

- same grade teacher discussion groups (planning; inquiry-based lessons; consolidation; next steps)
- sharing of grade-specific findings based on authentic student work samples
- school site discussions related to BIPSA planning for next year

Throughout the eight month duration of the project, the Numeracy Facilitator, Numeracy Consultant and two Numeracy Coaches were also available for each school to provide the teachers with on-going support in coaching, planning, modelling strategies, and/or providing co-instruction.

Methodology

Case study is an exploration or study of a bounded system which may consist of multiple sites within the same study. This research is an evaluative case study with the purpose of inquiry into an educational program in order to determine its effectiveness as judged by the researcher (Merriam, 2001; Stake, 1995, 2006; Yin, 2004, 2009). This case study has been informed by both qualitative and quantitative data and in this vein employs mixed methods of data collection. Ethical clearance to conduct data collection was granted by both Brock University Research Ethics Board and Niagara Catholic District School Board's research ethics boards. All participants (teacher participants, facilitators, parents/guardians of students) signed informed consent forms.

School Sites, Teachers and Students

In sum, three elementary schools participated in this program. Participating schools were chosen to build capacity within and throughout the system, as well as, staff readiness to incorporate inquiry-based professional learning in order to build capacity, inform instructional practice, and contribute to a culture of early years learning. The administrators at these schools held goals related to improving mathematics achievement in their school improvement plans – a program focused on early learning in math was anticipated to ameliorate for inconsistent student performance on the Grade 3 and 6 EQAO tests.

There were 16 teachers (ELKP, Grades 1 and 2) and 8 Early Childhood Educators (ECE) that participated in the program and their students indirectly participated as recipients of the teachers' professional learning. In total, there were 245 students in these 16 classrooms. The following **Table 1.** offers a coded profile of each of these school sites, the teachers, and their students. **Table 2**. is a summary of the student participant sample.

School Codes	Teacher Codes	ECE Codes	Grade	Number of Students
2	2AT	2AE	ELKP	22
	2BT	2BE	ELKP	19
	2CT	2CE	ELKP	26
	2DT	2DE	ELKP	21
	2FT		1	18
	2GT		1/2	16
	2HT		2	11
	2JT		2	12
3	3KT	3KE	ELKP	19
	3LT	3LE	ELKP	20
	3MT		1	10

	3NT		1/2	12
4	4QT	4QE	ELKP	6
	4RT	4RE	ELKP	11
	4ST		1/2	9
	4TT		1/2	13

Table 1. Summary of Sample: Schools, Teachers, ECE's, Grades, Students

Grade	School 2	School 3	School 4	Total Females	Total Males	TOTAL
ELKP (Yr.1)	35	20	11	35	31	66
ELKP (Yr. 2)	53	19	7	33	46	79
Grade 1	27	16	10	28	25	53
Grade 2	30	6	12	21	27	48

Table 2. Summary of Student Sample by Grade, School, and Gender

Data Collection

There were three sets of participants that data were collected from: teachers, facilitators and, students. Within each participant set, multiple forms of data were collected. This practice contributed to the triangulation of the data and the rigour of the findings.

Teacher and ECE Data: Surveys

In October, 2013 at the first plenary session, teachers were surveyed to capture their current practices and beliefs related to mathematics instruction. At the end of the final plenary session in May, 2014, the teachers were again surveyed to capture changes in their practices and beliefs as a function of the professional learning project. The surveys consisted of questions on a 5-point Likert scale (Strongly Disagree, Disagree, Undecided, Agree, Strongly Agree). Question items included statements summarizing contemporary approaches to teaching mathematics and commonly espoused teachers' beliefs about mathematics. The surveys were adapted from previously administered instruments (Foong & Perry, 1998; Perry, Howard, & Tracey, 1999; Perry et al., 2002; Perry, Wong, & Howard, 2006; Quillen, 2005; Van de Walle, Karp, & Bay-Williams, 2013; White, Way, Perry & Southwell, 2005; Wilkins, 2008). The surveys were coded for each of the teacher participants and question responses were compared for the October and May administration dates.

Teacher and ECE Data: Anecdotal Notes during Professional Learning Meetings

Anecdotal notes were taken by the researchers during the two plenary sessions and one of each of the half day sessions. During these sessions, the researcher was an unobtrusive observer taking fieldnotes of the professional dialogue and collecting artifacts. All three of the teacher, ECE and facilitator participants were observed.

Teacher and ECE Data: Interviews

Teachers (n=8) and ECE's (n=4) from each of the three schools were interviewed in May, 2014. These participants volunteered for this interview and were given release time. The purpose of the interview was to garner an elaboration of the educators' practices and beliefs with examples and illustrations from the classroom. Teachers were asked 8 questions about instructional strategies and

evidence-based instruction practices in mathematics. They were asked about their perceptions of students' understanding of math content and mathematics achievement. Finally, they were asked about their own professional learning growth and self-efficacy in mathematics instruction. The 30 minute interviews were transcribed by a transcriber.

Teacher and ECE Data: Learning Blogs

Teachers wrote learning blog entries periodically during the project. The intent of the learning blog was to track teachers' experiences and reflections throughout the project. The entries were open-ended and teachers were encouraged to journal about any salient experiences, observations, or questions related to their practice or student learning. This activity was meant to bring awareness to educators' connections to exemplary instructional practices. Subsequent learning blog entries were prompted by the facilitated professional learning sessions. The final blog entry was intended to be a comprehensive reflection on the project and what the teachers believed that they had derived from it. The confidentially coded learning blogs were collected by the researchers at the end of the final session.

Facilitator Data: Anecdotal Notes during Professional Learning Meetings and Interview

As noted above, anecdotal notes were taken by the researchers during the plenary sessions and at each of the on-site sessions observing the facilitator and the educators. Carlo, the K-12 Numeracy Facilitator was interviewed in May, 2014. The purpose of his interview was to garner an elaboration on his evaluation of the project and changes in teachers' and ECEs' practices and beliefs.

Student Data: Developmental Math Assessment (2013)

The student data were coded by: school/teacher/grade/student code/gender. Student participants were tested one-on-one in October and May on the *Developmental Math Assessment*. There were four forms or versions of the assessment administered to each of the developmental levels of students: Prekindergarten Age 3-4 (ELKP Year 1 students); Prekindergarten Age 4-5 (ELKP Year 2 students); Grade 1 Readiness (Grade 1 students); Grade 1 Level (Grade 2 students). Each form of the test had several sub-tests (see Tables 4-7 below for the list of sub-tests and results). Student performance was recorded on an nominal scale as indicated on the DMA as 0=None Correct; 1=Part Correct; 2=All Correct.

Data Analysis

The interviews (teachers, facilitator) were transcribed by a transcriber and the researchers then conducted qualitative data analysis including coding and collapsing data into themes. These subsequent themes were derived in response to the research questions. The learning logs and anecdotal notes were similarly coding using the same themes that evolved from the interview data. Interpretations of the themes were made and illustrative quotes were selected from all of the participants. These qualitative findings are mapped back to the five research questions in the following section.

The quantitative data (teachers' surveys, students' assessment data) were entered and analyzed using SPSS 19.0 (SPSS Software, 2011). The teacher survey data (October and May, 2014) and were compared using Paired Sample t-tests. The students' assessment data for pre-test (October) and post-test (May) were compared for all test items using Wilcoxon Signed-Ranks Test (Laerd, 2014).

Findings

The following section is a presentation of the findings based on the data analyses. These findings respond to the clusters of research questions related to: "Teachers' Practices", "Teachers' Beliefs and Attitudes", and "Students' Conceptual Understanding and Achievement". Within these three clusters, the findings will describe how the teachers: focussed on the relationship between the teacher, student and the content; identified curriculum connections; promoted students' understanding of mathematics content; and used instructional strategies. Teachers' beliefs about growth in their knowledge, self-efficacy, and intentions for future practice are summarized. Finally, the results of the analyses of students' conceptual understanding and achievement as a function of the evidence-based practices are offered.

Teachers' Practices

Are teachers focusing on the relationship between the teacher, the student and the content?

One of the first tasks completed in this project was to gather baseline data regarding students' existing knowledge of number sense. This in-depth information about each individual student, provided to the respective classroom teachers and early childhood educators, was a critical starting point for the facilitator's foundational position that educators' instructional decisions need to be data-driven. Upon reviewing this data, educators readily responded by noting achievement trends in relation to specific numeracy concepts and making instructional decisions accordingly. A factor in the teachers' approach to maximizing student achievement was to first to determine an accurate picture of students' conceptual strengths and weaknesses.

> I think that the biggest thing with the teacher and the student is for the teacher to see where the students are at. I think that was a key for them to understand, to really know where students' mathematical instruction starts, to know where on the trajectory or where on the landscape of learning the student is, to see the mathematics that the student is presenting. Often times the teacher would start a lesson because that is where they needed to start and not necessarily know why they are starting there, and I think that when teachers saw students' work it really became evident to the teacher (Interview, Facilitator 1F).

> When looking at my class data I was shocked that when my students were asked to orally count and count objects they had difficulty with it, the majority of the students could not complete this task. I was pleased to see that the majority of the class was able to compare and order and match. After looking at the data, this assisted me to find out the strengths and weaknesses of my students. From the data, I began to form my class TLCP where counting is the focus (Journal, Teacher 4RT, Dec 13/13).

> What was most valuable to us when we started this was we didn't know where the students were in math. We didn't even know where to focus. So the base line, the original assessment, really helped us focus on what we really needed to teach them and that's what we focused on. That's how this whole thing has actually helped us and the end result was that they did improve (Interview, ELKP Teacher 4QT).

> We have seen the students' testing and saw that some students improved and regressed in their math skills. With this information we, as teachers, can either give the students new challenges or continue to help with areas of difficulty (Journal, Early Childhood Educator 3KE, Feb. 25/14).

> Clearly a lot of work needs to be done with a focus on number sense, which is the key to understanding and allowing a progression to occur, however small or big it may be. Concepts of subtraction and addition need to be practiced ongoing (Journal, Teacher 3NT, Dec. 16/14).

> I was surprised at how many of my students were unable to answer questions about "1 more" and "10 more" but could answer questions about "less". I was also surprised to see how many year 2 students struggled with the question about matching. We have begun to use more/less more frequently throughout

> the day and also have introduced the vocabulary "match" into our lessons instead of just using "pair" (Journal, Teacher 3LT, Dec. 13/13).
>
> This year I really tried to take a few steps back and just really see where they are at and where I needed to move them individually, rather than as a whole. I tried to look at them more individually than I may have in the past (Interview, Grade 1/2 Teacher 2GT).

The importance of understanding the extent of students' conceptual knowledge included a clear impetus to honour what the child *does* know before leveraging students' prior knowledge to effectively address knowledge gaps. This moves a teacher's orientation of responding to assessment information away from a deficiency perspective focussed on what the student *does not* know.

> Today it was interesting to focus on the idea of what the child *does* know when looking at a piece of student work. When we really sat down to analyze student work without knowing who the child was or having any preconceived ideas of their learning, we were able to look objectively and see an accurate picture of where they were. It was interesting how first spending time on what they do know before moving on to next steps helped us to see learning in more of a positive light (Journal, Teacher 3LT, Jan. 15/14).
>
> Our meeting in January helped me to better understand that when I take a look at what the student *already knows*, then I can determine what should or could come next for that student. This is a great way to extend a student's learning so that they are challenged to go to the next level (Journal, Teacher 2DT, Jan. 27/14).

To foster the conditions necessary for dynamic inquiry-based classroom strategies, educators made a conscious effort to adjust their relationship with students to one that includes student driven practices. While this did not negate the need for some teacher driven instruction, it did provide the opportunity for students to take the lead in their own learning. Educators saw the benefits of allowing students to set the instructional pace and impact the focus of the lesson, which was made possible because the educators authentically listened to their students. Talking with their students and listening to them enabled educators to maximize their responsiveness to an individual student's learning needs.

> It is the talking and listening to the students as opposed to them listening to us. More team work... I guess what I found through instruction with the students is that the relationship definitely was really to listen to them and see exactly where they were at. Without listening to them, and having them reflect on what it is exactly that they were thinking, you really did not understand where their numbers came from. So, as a teacher, I definitely started learning to really listen to see at what level they were at and to see exactly what clues or pieces were missing with the student in order for them to get to the answer. That was the huge thing that I noticed. Before, without that, I would look at the end result and they [the students] were not able to decipher it either. So that was a huge piece that I learned that we needed, which is relationship with the student for sure in order to understand where they were coming from, or the fact they were coming from nowhere, if that was the case (Interview, Grade 1/2 Teacher 3NT).
>
> I think I've learned a lot in terms of shifting the way I've been looking at presenting concepts or the way that we relate to one another as I'm teaching. I try to take an inquiry-based stance... before I might have just introduced something and then we went and practiced it and it was more teacher directed and not really student driven. Now, this year, it's totally student driven and they often do the mini lessons, they often create the activities that the other kids are going to do, and go from there. We pull the curriculum in there, so it's like I'm covering the curriculum, or they are uncovering it, instead of me presenting it and saying, "this is what we are doing". So that's changed the relationship and my role in the classroom significantly (Interview, ELKP Teacher 2CT).
>
> You always keep the curriculum in mind and then if they're here you start there [where they are] and if they are moving fast you move fast. If they are not grasping it as fast as you thought they might, then you stay where they are and move as they go (Interview, Early Childhood Educator 4QE).

> I'm giving my students more of a chance to share their thoughts. So, if they suggest a strategy, we can look at it. I'll say, "Does that work for you?", "What works for you?" So that is what I noticed. They needed to share their own ideas, and I needed to let them. Then if something wasn't working, we'll go back and look at it again a different way... I'm pumping them up about it, saying that they can do it and it's okay if you made a mistake, and we can figure out what we did wrong. Then we go from there... I got them to sit down and think about what works for them... it's letting them lead some of the learning (Interview, Grade 2 Teacher 2HT).

> I have been reminded to allow children to learn at their own pace, and each child solves a problem in a different way or views it differently. There are different ways to reach a solution. Children enjoy learning when it is presented to them in a fun way and not in a typical mundane manner (Journal, Early Childhood Educator 3KE, Dec. 16/13).

Another expressed benefit of consciously listening to their students and fostering student driven learning was that it augmented the teacher's ability to plan activities and learning opportunities that used students' interests as a catalyst. It enabled educators to teach number sense concepts contextually, embedded into an activity or topic that was already of interest to the student, making learning opportunities relevant and engaging for the students. Such learning opportunities occurred because teachers remained flexible and responsive to students' interests.

> We have been focusing a lot on inquiry. It's based on the students' interests, and it's them [students] guiding us [teachers] with what's happening in the classroom when they go off to play and explore with things we have brought into the classroom. There's a lot of number sense is in there (Interview, ELKP Teacher 3KT).

> We did a counting jar over the year and students would bring in their own objects they would fill the jar. We would count them every day, we would estimate—how many do you think there are? We would count them and then we would sort them. So we grouped a lot of different math concepts into that. But it was stuff that they were interested in because it was stuff that they found from home so they were really excited about that. It was different to see. So say they had 40 Legos and then they had 40 little seeds. They would think that the bigger one is more—that there's more because it just looks fuller. Right? So we talked about that and that things can be different sizes. I guess they just weren't aware of that (Interview, Early Childhood Educator 2AE).

> We have to use a lot of visual ways to cover material. We have so many different ways because they are just in so many different places right now. We have a group that is just identifying numbers, so my ECE made up a little hockey rink, because they like hockey. Sometimes its basketball, we have different things. They have a little puck with a number on it, and then on the ice there's a number, and they just have to match it... One on one matching is needed for some of them (Interview, Grade ELKP Teacher 4VT).

> One day they were playing with money, so we learned a lot about money. I found songs about money. Patterning – we would look at patterning and the blocks that we had and then we would make necklaces out of the patterning, using fruit loops to make patterns and we found all kinds of things they enjoyed doing with patterning, even with clothing. And then they were identifying patterns in the room and they would make their own patterns out on the playground (Interview, ELKP Teacher 4QT).
> As educators we need to have a thorough understanding of the curriculum, how students develop, be aware of their interests and try and capture learning whenever and wherever it is happening in our classrooms, and provide opportunities and experiences that will make it happen. We need to see the students as capable, competent individuals who are able to grow (Journal, Teacher 2CT, Jan. 14/14).

Are teachers identifying curriculum connections and using curriculum mapping?

Curriculum mapping was a central element in one of the facilitated learning sessions, at which the facilitator had educators investigate the progression of curriculum expectations in number sense from primary grades into the junior division. This provided a framework for the educator's short-term instructional planning within a long-term context.

> With the teachers, they would see where the concepts were going, and understand that this is not an isolated concept within their grade and saw a connection between grades. We mapped all the way to grade 5 multiplication and where that starts in the earlier years. That was a huge opener for teachers to say that the concepts go beyond my grade, that this concept keeps going and where it starts, that a simple concept like counting was so important to concepts of addition/subtraction and then to multiplication/division. I think when teachers saw that connection, it was an important moment for them... Teachers would say, wow, this can lead to measurement and I see how the measurement connects to this concept in number sense, and I see how time and fractions or perimeter and addition are related. They had all these different connections that they start seeing as they worked through the year, so their short-term plans could connect with their long range goals (Interview, Facilitator 1F)
>
> After the release day while at the school, I realized that the Ontario Mathematics Curriculum expectations from grades ELKP to Grade 2 build upon each other each year. I found it purposeful to look at each grade's expectations and see the similarities and differences. As an ELKP teacher knowing what my students need to know by the end of ELKP and what the expectations are in Grade 1 etc. helps me to creatively plan and teach accordingly (Journal, Teacher 4RT, Dec. 13/13).
>
> Release Date Nov. 28: I found it interesting when looking at the way a student develops through different skills, such as counting. They first recognize one-to-one, then how do they count, do they understand once they have counted 5 dots on a dice face once it will be the same when asked again how many dots, or do they need to count the same 5 again.? Do they know that 5 items in a line left to right is the same as a line of 5 items organized top to bottom? This session inspired us to start a counting jar. Students can take ownership of it, count it at home, then share it with their friends the next day. Students can build their estimation skills, and counting manipulatives skills. During counting we can make piles of 5 or 10 to show them how it is easier/quicker to group them to count. We can then sort the objects after we have counted them (Journal, Teacher 2AT, Jan.18/14)

A profound realization of participants in this curriculum mapping exercise was that number sense is pervasive throughout the curriculum. Educators discovered that number sense was relevant to many other strands within any given grade, both within numeracy strands and evidenced in other subject's strands as well. This fostered the resolve that there was no need to teach number sense as a separate strand or as a discrete unit because number sense could be effectively taught by integrating its curriculum expectations into lessons rooted in other curricular strands.

> I always have in my mind what the [curriculum] expectations are that we have to cover, and base everything on that. If I can pull something from what they are doing, based on the expectations, I will do that, and relate it. Even if it is from a story, you can still get math from a story, like patterning, whether, again, its colors or numbers, anything like that. To me they are all kind of related (Interview, ELKP Teacher 4VT).
>
> It's just allowing those kids to explore and go according to their interests in what they want to find out. I'm asking them questions, "What are you doing here [with this activity]? What can you find out? How do you know this?" It's all making sure you know your curriculum very well so you can get those connections met within the curriculum. You're making sure you are hitting number sense... you may be working on 3D shapes in one area, but it's not focused just on the geometry of the shapes. You're bringing in that number sense from other parts of the curriculum as well (Interview, ELKP Teacher 3KT).
>
> It's not just teaching a strand and being done with it. It's connecting them. So, for example, when we learned time I'd say, "Can you see a fraction in that?" ...I mean you can't do it all, but it's exciting to see them making the connections and bringing it through. I'm trying to give them that time. That's been a really big thing for me this year (Interview, Grade 2 Teacher 2HT).
>
> I think it is totally integrated now, where it wasn't before. They are definitely integrating it. The integration piece is really happening... Once they see it and you lay it out for them, they can see how everything can link. But that's like me telling you "you can do it", well it does not necessarily mean it will happen. Whereas in here, because they have had the inquiry based learning experience and they are doing this math initiative they are saying "wait a minute I can link this". So they are naturally discovering it

themselves. And I think that instead of me telling you that you can do it, you have to discover it yourself that it can be done. I think they naturally go hand in hand that way (Interview, Vice-Principal 2V).

The ever present issue of time constraints on the teacher, needing to cover a substantial amount of curriculum within a discrete time period, was an issue that was identified by educators at one of the facilitated learning sessions. Teaching number sense in an embedded way was subsequently realized to be a solution to some of the expressed time constraints - it appeared to be a more efficient way to cover curriculum than teaching strands in silos.

When discussing curriculum expectations, one educators verbalized concern: "How do we move students forward when they're falling behind on a strand when we have this really difficult time constraint to manage? My students are all over the place on what they know and don't know in the various strands and expectations. Sometimes it feels like there's not enough time to do everything (Fieldnotes, session 2).

I think that one thing that I learned as a teacher is that number sense is integrated everywhere. I think prior to this, and I think I am not going to speak for most teachers, but I think we kind of did it in blocks, like a unit, and then you put it aside. But I am... interweaving it... am revisiting it ongoing. So it is kind of like a weaving it in and out, and you see it everywhere now. And I think the students really have made that connection that numbers are everywhere, like even when you are looking at a pattern or what have you, you are always bringing those numbers back in, which for me was a big eye opener. So embeddedness versus a discrete unit - definitely. It is keeping number sense open at all times... *Before it did not seem I had enough time to teach but now it seems that it is integrated so you have more time for some reason*... It seems like you take that [number sense] unit out and are covering it everywhere. Whereas before you might have taken that month for just number sense, now you are just spreading it on top of another layer (Interview, Grade 1/2 Teacher 3NT).

Are teachers promoting students' understanding of math content? (beyond rote learning)

The importance of focusing on students' process, not just the product of their work, was identified as an avenue for promoting students' understanding of concepts beyond rote memorization.

I enjoyed reading and discussing Pedagogical Documentation and reflecting on the practise of looking at the importance of the process rather than just the end product. Realizing that learning becomes more meaningful when we listen and allow students to take the lead in the learning and making them a part of the learning process, providing them with ownership of it (Journal, Teacher 2CT, Jan. 14/14).

When discussing the importance of helping students understand the process of a task, one teacher shared her reflection with the group: "With the kids I show them each step, and work with them on each step... then after a few steps they have figured it out and they don't need my help anymore. After that I see them excited to show their friends the steps they used, if their friend gets stuck on a problem" (Fieldnotes, Session 3).

Math is no longer *'this plus this equals this and looking at that end result* '. It doesn't really matter about the end result, it is just how we get there in our travels (Interview, Grade 1/2 Teacher 3NT).

When we dug deeper into the area of pedagogical documentation teachers really started to see the importance of the process of learning and not the product of the learning (Journal, Facilitator 1F)

If the process is to be valued, teachers identified that having students explain their thinking and talk about the strategies they used to arrive at an answer reinforces the importance of processes. Further, it develops a greater understanding of the material covered as students have to explain "why" and "how", which appeals to higher-order thinking. One by-product of this approach is that teachers are using fewer worksheets in favour of more meaningful collaborative strategies. Another benefit is that this higher-order dialogue renders a student's understanding of any particular concept more transparent to both the teacher and the student.

> Sitting down with the child to ask them about their thinking added another piece to the puzzle which can then be followed up on. In my own class I am going to try to document more using pictures, and also using small labels that I can stick on their individual number sense page to write down observations we see during free play (Journal, Teacher 3LT, Jan. 15/14).
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> I did not have as many worksheets this year, I did not have them do as many workbook activities this year. This year it was more or less problem based learning, and them looking at, "What are we really talking about?" and "What does that mean?" rather just doing a question in a workbook to figure it out (Interview, Grade 1/2 Teacher 2GT).
>
> I'm asking the students, "How do you know that? How are you getting that?" I'm letting them figure it out for themselves... I'm not drilling... I'm telling them, "You can figure that out for yourself". They need to find out and explain their thinking, instead of memorizing... They need to figure it out for themselves. They need the hands on manipulatives or activities to be able to figure out their answers for themselves. They really feel so good about themselves when they can do it on their own. You know, when you are observing students learning they feel proud when they are able to verify what they thought (Interview, ELKP Teacher 3KT).
>
> I know I say, "Well how do we know?" instead of saying, "Did you make a mistake?" and "Let's try it again and let's try it like this". I've noticed a couple of kids do that [say "how do you know?"] on their own now (Interview, Early Childhood Educator 2AE).
>
> It was interesting to be given a math problem and see how many different strategies people use to solve the exact same problem. It really makes you become aware of how important it is to allow students to share the strategies they used to solve a problem not just share the answer. That way all the students benefit from seeing how problems can be attacked in different ways and then start to identify the most efficient and effective strategies to use. The opportunity for sharing also helps students consolidate their learning. When the student voice is valued then a community of learners is created and students can become more comfortable in taking risks and make attempts to transfer their learning to a variety of situations (Journal, Teacher 2CT, Feb. 20/14).
>
> First I think when [the facilitator] came in and started teaching us a couple of those strategies... it just kind of opened our eyes a bit to see how the children see those numbers. I started doing those from the book as well, and I did my own cards. The children would come up and show me how they got that number and it was interesting to see the children counting by ones, and some of the children would be grouping, and then some of the children would say that they saw what the other child was doing. I did see that growth, so those strategies were helpful for me, to see how those children were looking at numbers, or what their understanding was, and what they were not seeing. So it shows you what exactly what you need to work on with them and even how important it is to do that each day, and it has to be all year long (Interview, Grade 1 Teacher 2FT).
>
> An educator shared: "When we let the When we let the kids talk things out with each other, it raises their self-efficacy because they have someone to work with so they'll take a risk, rather than saying they can't do it & shutting down. We let them know the *process* is the important thing, not the *final answer*." (Fieldnotes, Session 3).

In order to model abstract thinking processes, and subsequently assist students to communicate their higher-order thought processes, educators need to demonstrate their own thought processes by communicating their thought processes audibly. Math talk needs to be embedded into the culture of the classroom.

> During a facilitated learning session, the facilitator emphasized that it was important to talk it out, to talk out loud when you're working on a problem so you're modeling the thinking that you go through for each step. This overt modeling helps the students to catch on faster to productive and successful strategies. An educator attending the session affirmed, "Yes, they need to hear my own thinking processes, and then you start to hear them using the same words and phrases." (Fieldnotes, Session 3).

> Even what's on their bulletin boards in the halls, you can see that there is a lot more math talk. It was interesting because I can see the excitement in them, they come and share things and they say you've got to come and see and what they did (Interview, Vice-Principal 2V).

Several teachers affirmed the importance of making math come alive as a worthwhile approach to supporting deeper levels of student understanding and achievement. Manipulatives and visuals were perceived as successfully supporting students' conceptual understanding because it fosters student engagement and makes abstract concepts more meaningful and relevant. How the teacher brings math to life varied by classroom, but a common element was to appeal to students' kinesthetic preferences rather than using more traditional paper and pencil methods.

> We had to find ways, and we found ways, of trying to make that [number sense concepts] come alive and act those things out, and then they could see it and they know it. We did a lot of activities everyday like standing on the carpet and we would count how many people are here. I'd say, "Ok we are going to send 2 to the bathroom so how many people are left? What if we bring 4 from another class, how many are left? So we did a lot of that, but it *had* to be hands-on visual kinds of things. A lot of it was taking what's on paper and bringing it to life... Paper doesn't work for us, ever (Interview, ELKP Teacher 4QT).

> I think that with anything you learn, not just with math, but it needs to be relevant or you don't remember it. It needs to be something that is meaningful to them, that they're interested in so lots of times they'll go to something that they're already interested in and they're already doing and I'll just add the math in there. Not really add the math in there, it's point out the math that's already there because math is everywhere. I mean you can walk into my room at any point of the day and you can find math, so it's more like helping them recognize 'oh I'm using this skill' or making them aware that that is what they are using. It's helping them build on to what they already know and I can take it a little bit further to consolidate something, or maybe change their understanding of something, or maybe they misunderstood something. If they are doing it and interested in it, that's a connection for them rather than it being just a photocopy (Interview, ELKP Teacher 2CT).

> Our kids grasp more with visuals, with their hands, rather than saying let's do a number sentence... gauging their interest in what they enjoy and just putting the math in there because if you said "math", our kids wouldn't know what you are talking about. I would just say that we don't focus so much on the memorization of the math itself, but just kind of putting it into everything in their daily life as a whole. They're getting the math and they're learning skills they need to get (Interview, Early Childhood Educator 4QE).

Are teachers using instructional strategies?

Several teachers reported using inquiry-based strategies. Their commitment to inquiry is fueled by their observation that it supports students' knowledge of multiple ways to complete tasks and to solve problems, it helps students to make meaningful schematic connections, and it keeps students engaged in the learning process. Further, both small group and large group classroom, discussions are facilitated by the teacher's interaction with students working together on inquiry activities.

> Play based inquiry allows children to learn at their own pace and see that there is more than one way to reach the same answer (Journal, Early Childhood Educator 3LE, Dec. 16/13).

> Today we discussed the gap in student learning from grade 3 to 6 of about 15%. After having the opportunity to reread Pedagogical Documentation, it was interesting to see how much I took away from it again. After reading, we discussed what we thought to be of concern within our school being students' lack of schema (background knowledge) as they do not have the experiences necessary to make good meaningful connections. We continued to discuss how pedagogical documentation is ongoing which facilitates growth and improvement in student achievement. In our primary classes we try to look for opportunities to extend student learning through inquiry. We also have used a math strategies wall where

> students can use it as a reference for number sense and numeration strategies. We will begin to use learning stories to demonstrate student growth (Journal, Teacher 3MT, Jan. 27/14).
>
> The inquiry part of it has made a big impact on those kids that are not always on task because they do get interested in it, they are intrigued by things. It's fun! (Interview, Grade 1 Teacher 2FT).
>
> I think it's just a lot more hands on easier for them grasp. You are not just sitting at your desk or the computer doing a work sheet... It's a lot more hands on and we're using inquiry... so we're not just sitting here getting bored with my pencil and my paper. It's [number sense] embedded in all aspects of it so I think that just makes it a little more real for them and easier to grasp (Interview, Early Childhood Educator 4QE).
>
> We play and explore in the morning, and we have a lot of large group activities afterwards... So, if I can get in there, and start manipulating with them... then I find they are a little more open with me... We work together, and talk about it together, so there is not just small group thinking about it. Everyone gets involved so that the thinking gets a little bit higher, because there are more people involved. Their process of thinking is helping those other ones explain more and understand a little more (Interview, ELKP Teacher 3KT).

Teaching through problem solving with access to manipulatives was perceived to be an engaging method to effectively put skills into a context where students reason their approach based on the elements inherent in any given problem they are solving. This provides a context for fostering independent thinking within the framework of peer support. In the example below, access to manipulatives helped make the appropriately challenging problem more accessible to students.

> When they [the students] are talking together or they are problem solving together, it's just that team work, and it's like, "let's try and figure this out". Like the other day, they wanted to figure out how many pennies were in this big bag. I thought *that will take forever*, but then there were three students deciding. First they started with groups of five – and they thought this will take a long time. Then they did groups of ten, so they ended up getting to 697 pennies, in about 10 minutes or so. That was neat to see. They were pretty excited. So it's that excitement... They figured out their own way which is automatically more engaging. And that's where you bring in all that number sense and you sit down with them and say, "we have a 100 here where else have we seen 100?" You are looking at all these different frameworks (Interview, Grade 1/2 Teacher 3NT).

One consideration regarding access to manipulatives and rich resources is the inequitable availability among schools. One educator expressed the sentiment that enthusiasm for a particular strategy is mediated by one's ability to access the manipulative at one's school. There was some variance among schools regarding the existence and availability of various specific manipulatives.

> In the Math Inquiry Session of January 2014 it was mentioned... to not worry about what is not available and to concern yourself with what is available. I know that I do worry about not having resources available and thinking "Well, how am I supposed to implement certain things when I don't have it?" With this thinking I become stressed and feel like I am not doing my job adequately. I now have to learn to use what is readily available to meet students expectations and maybe it will spark other ideas to other resources to help with both students' learning and my teaching. I do realize that this will take time and it will become easier (Journal, Early Childhood Educator 3KE, Jan. 17/14).

While there appears clear support for, and enthusiasm for, implementing evidence-based practices in the classroom, it can be difficult for teachers to know the extent to which they should be using such practices. This would be a viable item for further discussion at future sessions conducted by the facilitator. Although it isn't possible to have a concrete way of definitively declaring the percentage of class time that should be allocated for such practices, a strategic discussion about this quandry, where colleagues can share personal perspectives and approaches to related best practices, may be helpful for the emergent implementer.

> Our last visit (visit #2) in January we discussed documenting the students journey in math. My concern for this and in my classroom is that I am wondering if I am providing enough opportunity for math activities

> during play and for enough math talk between myself and my students? (Journal, Teacher 3KT, Feb. 25/14).

Issues pertaining to the optimal assessment of students' knowledge who have learned concepts via evidence-based practices, such as inquiry, is an important item for consideration. Interactive dialogue with the student was highlighted as an important vehicle for cultivating an accurate picture of students' understanding. In the same way that interactive activities, rather than worksheets, assist students to develop their conceptual understanding more deeply, teachers' use of interactive and varied assessment practices will glean a more multi-faceted picture of what students have learned. Hence, if concepts are taught in an interactive way, there needs to be consideration of gathering measures of student achievement or understanding in interactive ways, as well, which includes multiple opportunities for students to demonstrate knowledge. The challenge of documenting numerous and varied assessments of student learning was also expressed.

> Definitely we need to look at what students do and don't know on a piece of work, however as teachers we need to verbally ask questions to reveal more information because at times, students have thought abstractly or have hidden knowledge that we cannot see on paper (Journal, Teacher 3NT, Feb. 21/14).

> It is important for us as professionals to take great care in our reflection and assessment practices to accurately gain a true picture of where each individual student is, where their needs are and how they can be moved forward to meet attainable goals successfully. Looking at methods of how we assess and keeping them specific and relevant, with much opportunity for students to demonstrate their knowledge, is key to successful teaching practise (Journal, Teacher 4TT, Jan. 29/14).

> The article given to us to read (Pedagogical Documentation) said that there is no one right way to document learning. The point is to document so that you can support student learning. Good to know but documenting is very difficult since there are so many students learning on so many different levels all day long. It is overwhelming and can be time consuming. That is why I like the other part of the article that says to "establish your primary purpose". I have a better understanding of what I should be doing but I am still on the road to "discovery"! (Journal, Teacher 2DT, Jan. 27/14).

Teachers' Beliefs and Attitudes

Is there Growth in Teachers' Knowledge of Mathematics Content and Instructional Practices?

At the end of the project, seven questions on the teachers' survey, *Beliefs about Mathematics, Mathematics Learning and Mathematics Teaching*, had significantly changed response patterns for the teacher participants. When all educators' data were analyzed in an aggregate fashion, there were four questions with significant results; when the Early Childhood Educators' data were separately analyzed there was one question with significant change. This survey of their beliefs was on a Likert-scale (1=Strongly Disagree; 2=Disagree; 3= Undecided; 4=Agree; 5=Strongly Agree) and consequently the means (M) reflect these values. **Table 3.** is a summary of these significant survey questions, statistics, and interpretations.

Survey Questions	Paired Samples t-Tests All Educators	Paired Samples t-Tests Teachers	Paired Samples t-Tests ECE	Interpretation of Significant Results
Mathematics is computation.	t(20)=-3.77, p=<.001, CI=[-.51, 1.77] T1 ($\bar{x}$ = 3.05) is greater than T2 ($\bar{x}$ = 1.90)	t(11)=-3.45, p=<.01, CI=[.27,1.23] T1 ($\bar{x}$ = 2.58) is greater than T2 ($\bar{x}$ = 1.83)	t(7)=-4.43, p=<.01, CI=[.99,.3.26] T1 ($\bar{x}$ = 3.8) is greater than T2 ($\bar{x}$ = 1.75)	All Educators, Teachers and ECE's were less likely to agree with this statement at the end of the school year.
Right answers are much more important in mathematics than the ways in which you get them.	t(20)=-2.17, p=<.05, CI=[.02,.75] T1 ($\bar{x}$ = 1.67) is greater than T2 ($\bar{x}$ = 1.29)	t(11)=-2.35, p=<.05, CI=[.02,.65] T1 ($\bar{x}$ = 1.42) is greater than T2 ($\bar{x}$ = 1.08)	X	All Educators and Teachers were less likely to agree with this statement at the end of the school year. NOTE: ECE's means were similar at T1 and T2
Mathematics learning is being able to get the right answers quickly.	X	t(11)=-2.80, p=<.05, CI=[.09,.75] T1 ($\bar{x}$ =1.75) is greater than T2 ($\bar{x}$ = 1.33)	X	Teachers were less likely to agree with this statement at the end of the school year. NOTE: all Educators and ECE's means were similar at T1 and T2
Periods of uncertainty, conflict, confusion, surprise are a significant part of the mathematics learning process.	X	t(11)=--2.57, p=<.05, CI=[-.93,.-.07] T1 ($\bar{x}$ = 4.25) is less than T2 ($\bar{x}$ = 4.75)	X	Teachers were more likely to agree with this statement at the end of the school year. NOTE: all Educators and ECE's means were similar at T1 and T2
Being able to memorize facts is critical in mathematics learning.	X	t(11)=-2.72, p=<.05, CI=[.18,1.66] T1 ($\bar{x}$ = 3.00) is greater than T2 ($\bar{x}$ = 2.08)	X	All Educators, Teachers and ECE's were less likely to agree with this statement at the end of the school year. NOTE: all Educators and ECE's means were similar at T1 and T2
Mathematics learning is enhanced by activities which build upon and respect students' experiences.	t(20)=-2.96, p=<.01, CI=[-.649, -.113] T1 ($\bar{x}$ = 4.33) is less than T2 ($\bar{x}$ = 4.71)	t(11)=--2.35, p=<.05, CI=[-.64,-.02] T1 ($\bar{x}$ = 4.42) is less than T2 ($\bar{x}$ = 4.75)	X	All Educators and Teachers were more likely to agree with this statement at the end of the school year. NOTE: ECE's means were similar at T1 and T2
I know how to effectively monitor mathematics activities.	t(20)=-2.68, p=<.01, CI=[-.847, -.106] T1 ($\bar{x}$ = 3.71) is less than T2 ($\bar{x}$ = 4.19)	t(11)=--2.97, p=<.01, CI=[-1.16,-.17] T1($\bar{x}$ =3.58) is less than T2 ($\bar{x}$ = 4.25)	X	All Educators and Teachers were more likely to agree with this statement at the end of the school year. NOTE: ECE's means were similar at T1 and T2

Table 3. Summary of Significant Results of Teachers' and ECE's Survey

Based on these significant survey question responses, educators are less likely to hold the belief that mathematics problems are solved by proposing an absolute solution – there are multiple pathways to process mathematics questions. Similarly, these educators are more likely to now believe that mathematics is a dynamic of many different ideas and learners interpret and organize this dynamic of information - this is part of the learning process in mathematics. In general, educators are less likely to hold the belief that it is important to memorize facts in mathematics. These educators do not contend that mathematics learning is demonstrated through computations and the ability to memorize facts, procedures or formulae. In a similar vein, these educators do not believe that students should be focused on quickly getting a correct right answer to a mathematics problem – the process of problem solving and understanding why and how one derived at a solution is of great value. The educators regard a mathematics learning context as one that is enhanced by challenging mathematics problems within a supportive environment, and activities which build upon and respect students' experiences. The educators now appreciate their key role in monitoring and being responsive to students' activities in mathematics.

The concept of a 'growth mindset', versus a 'fixed mindset' was a point of discussion at a facilitated learning session. Whereas a fixed mindset limits improvement in math because of perception that math is a gift and one either has this gift or not, a growth mindset advocates that knowledge can be impacted with effort. Accordingly, the facilitator cautioned the educators against emulating a fixed mindset, where mistakes tend to be stigmatized rather than being perceived as a welcome opportunity for learning. If effort can promote learning, then mistakes become a natural part of a healthy learning process. Educators who foster a growth mindset in their students are cultivating resilient learners and promoting greater opportunity for achievement. Educators embraced this concept, and strove to embody a growth mindset in their classrooms. This was a new approach, for some, and represented growth. Accordingly, educators are now more likely to accept that periods of uncertainty, conflict, confusion, surprise are a significant part of the mathematics learning process.

> I am thoroughly enjoying exploring the principles of counting and the major concepts of number sense and numeration within the context of inquiry based learning. The learning trajectories and developmental levels really provide a great resource for supporting student learning. I believe in the "growth mindset" we all need to have so we can all become persistent and resilient learners who celebrate the learning, use our mistakes as moments to problem solve and grow and celebrate and value the process and effort not always just the final product (Journal, Teacher 2CT, Dec. 12/13).

> I found the session very interesting and full of information. I liked to see how we all can look at a problem differently and solve it in different ways. Some things that stuck out were that it's always about learning, making mistakes and keep learning keep trying. It's ok if it's hard to solve, just keep learning. Mistakes are a natural part of learning. Don't have a fixed mindset, effort doesn't mean you aren't smart! (Journal, Teacher 2FT, Dec. 13/13)

Inherent in a growth mindset is an explicit recognition that there are multiple ways to tackle a problem or task, which values process over product. Further, this conceptual perspective includes an openness to multiple ways of thinking and an acknowledgement of the viability of multiple methods to solve math questions. Many educators cited this as an aspect of their growth in their instructional approach. Therefore, educators are now less likely to view mathematics learning as being able to get the right answers quickly. They now are also less likely to feel that right answers are much more important in mathematics than the ways in which you get them. Accordingly, they are less likely to perceive that mathematics is computation.

> This opportunity gave me the chance to see how many different ways there are to figure out a solution to a problem. As a teacher, giving students those learning opportunities to "play" with numbers was a powerful moment and I left the meeting ready to observe the many ways my students see and "play" with numbers (Journal, Teacher 4RT, Dec. 13/13).

> Great in-session with a lot to leave with and to think about. What I have been discovering, the more that I do go along, is that students, when it comes to math, definitely do progress at their own rate and in their

> own methods. In the classroom we need to offer a wide range of materials/manipulatives in order to let the students explore in their own ways. Through the use of "play based inquiry" this gives the students an outlet and an opportunity to explore in their own ways and to understand in their ways and at their own level of readiness. As educators we need to be open to ways of thinking to allow the students' natural progression of understanding to happen at any points of the year (Journal, Teacher 3NT, Dec. 16/13).

> Well I think when you are working with the students, before I would say, "This is how you do it", "Here is the correct way", or "No that's wrong". But when you are letting them talk to each other and work in groups, they are solving it. Some are making mistakes and then they are talking about it with their peers and they are showing them saying, "This is what I was thinking", or, "Oh, look how someone else solved it" and they get excited about it. It is not coming from me it is coming from them. They are talking with each other, and they are comfortable with each other. If I asked them a question sometimes they will get nervous like, "Oh no, the teacher is coming" but when they are in their group, they open up, and it takes away some of those barriers so they are able to talk to each other help each other out... There is a lot to be gained from their mistakes and looking at how other kids thought about it (Interview, Grade 1 Teacher 2FT).

The importance of making math dynamic and contextual was an area of growth cited by some educators. The benefit of how this interactive approach supports student achievement fueled educators' immediate application of this new approach. Accordingly, educators are now less likely to contend that being able to memorize facts is critical in mathematics learning. Educators are now also more likely to advocate that mathematics learning is enhanced by activities which build upon and respect students' experiences. Educators' experiences implementing dynamic strategies in their classroom provided the impetus for their confidence that they now knew how to effectively monitor mathematics activities.

> I learned a lot. Take it off the page, I think that's the most important thing to have. Take it off the page, make it come to life so that they [students] see it and they can do it. If they can do it, they can learn it (Interview, ELKP Teacher 4QT).

> We did a debrief when we read the book "The Ten Flashing Fireflies". We got this big plastic jar and ten little toys to represent the ten flashing fireflies, so as we were reading the book we would ask the kids, "so how many do you think are in here?" Then they'd say "let's count them, let's do it in real life"... So they got to listen to it, see it in the book and then act it out. It was just compounding itself and making it a little more concrete for them... Here, we do a lot more hands on now and I think it's easier for them to grasp because you are not just sitting there with your pencils writing your number sentence (Interview, Early Childhood Educator 4QE).

Ultimately, educators welcomed the opportunity for growth, as they recognized they were at a critical inflection point on their own learning curve. A receptivity regarding professional growth is an important element in an educator's ability to cultivate an updated, effective repertoire of high quality instructional practices. Due to a clear correlation between instructional quality and student achievement, educators recognized that in order to promote student achievement they needed to invest in building their personal capacity.

> My class had similar areas that we needed to work on as other classes at my school and with others. Place value and counting, and more and less were areas to focus on. I found that out myself when working with the students in the class. I was looking forward to learning new strategies to help my students pick up those concepts because I was frustrated as well (Journal, Teacher 2FT, Dec. 13/13).

> Today's session went well. We read the monograph: Maximizing Student Mathematical Learning in the Early Years. I found some areas to be interesting, particularly the subheading (Deepening of Teacher Understanding + shifts in instructional practice = impact on student learning), as we are learning to be flexible in our teaching practices as there is research to show that early math conversations and knowledge have long lasting implications for later school achievement. My concern at this point was that some of my students do not have the prior knowledge and experience they bring to school therefore at times it has become challenging. For example, at this point in the school year I have a student who still

finds it difficult to count orally by 1's consistently and accurately. How can I move forward with having this student understand place value for instance (an area of concern for many in my class) if my student is unable to understand counting? I guess it's a learning curve for all of us teachers on this journey as well. I need to look at other ways this student can develop these necessary skills (Journal, Teacher 3MT, Feb. 25/14).

I think that teachers had lots of growth because they were open to the learning. They were at the point where they were saying "I just don't know math that well, I don't know the content that well", so I think that they were open to the idea that there was content to be learned (Interview, Facilitator 1F).

Accordingly, educators perceived their new instructional strategies and practices discussed at facilitated sessions to be of value to their instructional repertoire. Some self-reported growth was identified specific to a concept, such as teaching students how to compose or decompose numbers. Other educators identified growth that was more broad in scope, such as skills relevant to creating and monitoring open-ended activities, how to apply differentiated instruction when problem solving, or how to promote a student-centered learning environment.

I feel the release day offered me new ways to show children how to compose and decompose numbers (Journal, Early Childhood Educator 4QE, Jan. 7/14).

Our facilitator used a great minds on activity which involved solving different math problems. It allowed me to see that there are more effective ways to solve the problem then the way I was taught using standard algorithms (Journal, Teacher 3MT, Dec. 13/13).

I think that my content knowledge is probably the same, but my ability to teach different strategies or to recognize different strategies has definitely grown, which has helped me, and in turn helped my students (Interview, Grade 1/2 Teacher 2GT).

In today's session, I have learned the *5 productive talk moves* stated in the reading material that was given to us. This will help me to help the students to discover their thinking process and to amplify it (Journal, Early Childhood Educator 3KE, Feb. 25/14).

Letting the kids lead, that was a big change for me. Standing back and letting go. We have done it for so long in Literacy instruction, it took some time for Numeracy to catch up... I am still learning but there needs to be a willingness to learn, and then you will be ok. You just have to try it and if you're not successful at it the first year it doesn't matter as long as you are open to it. But I think how they looked at numbers or how they learned is a big shift for me. Letting the students talk and almost letting them learn from each other... I am happy that I am learning. I am happy that I am not a stick in the mud not saying, "Nope, I am doing the old stuff", because I do see that I am happy and that I am open to this. I am learning how the kids are learning, they are more engaged and I feel more successful (Interview, Grade 1 Teacher 2FT).

The things that I've seen them create and explore... It was like I was limiting them before with my very close-ended activities and I wasn't really reaching anybody's potential because the activity was so restrained. Now that they have the freedom to do it, they can show me in so many ways that their skills are so all over the place (Interview, ELKP Teacher 2CT).

Is there an increase in teachers' self-efficacy in mathematics instruction?

In their quest to improve or update their instructional capacity, educators appreciated having access to support, both human support in their classroom to model best practices, and access to tangible resources. Support was a positive influence in their efforts around instructional capacity.

I think this is a great opportunity for all involved and it is key that we are all on the same page. I like the fact that where I am lacking in this area, I will have support for a better understanding, and with the combined knowledge of the children's interests and developmental levels, success will be achieved (Journal, Early Childhood Educator 4RE, Dec. 28/13).

> I enjoyed seeing the data and seeing how some/most children improved and where we still need to focus on. I will check out the web site EduGAINS.ca [a website discussed at the facilitated learning session] for ideas and guidance. It is good to have strategies presented that you maybe haven't thought of. It was interesting to see how wording can determine the outcome of the answers, the understanding of what is expected. Thanks for the insights (Journal, Early Childhood Educator 3LE, Feb. 25/14).

> Out of all this program, I liked having people come in [to my classroom]. I like to learn those things and I like the support. Really, the support is big because you can *tell* somebody, "Ok, this is how you are doing it" and it goes on a shelf to be honest because if you don't understand it you are not going to do it. But if you have someone coming in and showing you, demonstrating, modeling, and you're going to another class to see it - if you have that then you are going to use it. So I just hope that they do something every year, it just helps, otherwise you just take something and it goes on the shelf and that is a waste.... You need instruction. You need people to come in to a class and show you if you want it done... Not everyone can learn by looking at a book, you want to have someone who knows about it come in a show you, and then you can try it, and you are a little more comfortable to try it. And then they can say, "try this" and if everybody else in your hallway is trying it, then you are learning together (Interview, Grade 1 Teacher 2FT).

Talking with other educators and having the time to dialogue around best practices was a specific form of support that had a positive impact on educators' confidence. These opportunities for dialogue provided a welcoming environment in which one could show vulnerability by asking questions without being stigmatized. Self-efficacy needs to be sufficiently robust in order to risk such vulnerability, and involvement in the project appeared to foster such confidence.

> My confidence totally gained throughout the year. Even just meeting with our groups and sharing stories, you get ideas from some of the other teachers and then you can bring it back into the classroom... We would have liked to see a little bit more of hands-on modelling in the classroom [from the facilitator]... I don't know if it's available out there, but ... it's always easier to see something demonstrated and say, "oh yeah, I can do it that way"... Like the one time we went into one of the classrooms, we had the manipulatives and all of us were in there and we were doing an activity and it was kind of nice to see the hands-on demonstrated and then we can bring it back into our own classroom (Interview, Early Childhood Educator 2BE).

> The Collaborative inquiry... talking to the other [participants], talking about "What have you been doing?, What's working for you?", and when we had the last meeting, on the 20th with [participants] from other schools, that was really helpful because they talked about some things they implemented themselves that I have already talked to my next year's teaching partner about doing in September (Interview, Grade 2 Teacher 2HT).

> I think I have learned a lot, I would definitely have to say that. I also learned we don't really know enough about math as well, although it is grade 1/2 there definitely has opened our eyes in just different ways to approach something and it is like that in every subject. We don't have all the answers but it is just realizing that, "ya we don't have all the answers", and it is up to us to jump in and ask those questions. Definitely is has been a welcoming environment for us to actually ask questions. But you have to be willing to realize that in order to ask those questions and realize that there is still so much to learn about math (Interview, Grade 1/2 Teacher 3NT).

Educators appeared to be more confident in their approach to how they were covering their curriculum. Despite the consistent pressure of time constraints on classroom activities, they began covering select material more thoroughly because they gave them self permission to trust their instructional instincts enough to slow down and cover material more thoroughly when needed, rather than succumb to rushing through curriculum to be covered. Less stress was experienced around time constraint issues.

I feel better, I feel more excited, I feel more like I am doing it [number sense] justice... I know that I am not just skimming the surface of all these things, I am doing a much better job with those topics now (Interview, Grade 1/2 Teacher 2GT).

I can admit to them that I don't know everything. I struggle too, but that's ok, you work through it. And just slowing it down. Where before I was so worried about producing, where now I want to take that time with them... I need time to sit and talk with them. Where I didn't do it in all areas before, now I want to, it's more important for me. I get a better grasp of who they are. It's [being involved in this project has] given me the confidence to do that, to know that's ok. It's ok that I don't have 15 different items that I've marked and recorded. That doesn't matter. You know you have those few good ones, more in depth ones, and that's what matters (Interview, Grade 2 Teacher 2HT).

I don't have to sit down as a whole class and say, "Ok, we are going to do adding and subtracting today, or we are focusing on patterns today". I know that I am going to reach those strands everywhere in my inquiry classroom. It's popping up everywhere and it's based on the children's interests. I don't have to stress all the time because now I know I'm reaching [covering] the strand. I know that within the little groups or talking one on one that I am able to address that math curriculum... I remember teaching last year, it was June and I was thinking, "I haven't done adding and subtraction yet". This year we don't have to do just a month of adding and subtraction. We are always talking about it everywhere... I'm not worried about what strands I have covered or haven't covered. It matters how many kids you have reached with that learning, like with your assessment and all of that documentation. So that's something I would have learned, and I was thinking, "Why didn't I make my life easier before?" (Interview, ELKP Teacher 3KT).

What is the impact on teachers' intentions for their future practice?

Educators saw value in understanding where students were in their learning process early in the school year in order to move student achievement forward in a strategic manner. Ideally, this would involve comprehensively assessing students in the Fall, and then using an approach grounded in the stages of development and the learning trajectories to guide their instructional pacing in order to strategically improve individual students' knowledge. Prioritizing student learning around the 8 principles of counting was another element that educators would now use to make more informed instructional decisions from the onset of the school year.

Another thing that they've voiced strongly is the diagnostic math assessment being administered in the Fall. I have been giving them some assistance with that, perhaps the supply teacher coverage or something so that they can get a handle on it from the beginning of the year. Some have said that a mid-year assessment might be a good too, and then we could target areas (Interview, Vice-Principal 2V).

I was excited to learn about the stages of development in the different areas of math. Can't wait to use this to find out where the students are and then teach next steps. Although we teach some of these skills naturally, there are a lot of little steps in between that will help us teach to the next level without guessing or relying on old strategies. It just makes so much sense. I wish we had access to this sooner and I can't wait to make use of this information in the class (Journal, Teacher 2DT, Dec. 13/13).

In addition, we discussed the 8 principles of counting which was interesting to learn that we assume students know the simplest counting activity, but may not. For example I assumed it would be easy for my students to know one to one correspondence but came to discover some may struggle with understanding that each object being counted must be given one count and only one count. They can do this easily by moving one item out of the way after it has been counted. I really learned a lot from the learning trajectories. I need to focus on the developmental levels for counting before I can move along. This will be my focus going forward (Journal, Teacher 3MT, Dec. 13/13).

Today during the release day it came to knowledge how different students' thinking is for many different skills. 'When we looked at the simple math questions (addition/subtraction/ multiplication) and had to figure out how the student solved their problem. It was interesting to hear what their thinking process is, we are so used to using the basic carry a number, and so forth, the standard way. We were given the 8 Principles of Counting. Again this was interesting, I was unaware of any principles. We just assume

students know when they count how many objects are in a set, they can tell us how many are there, or so forth, but some cannot answer how many are in a set, right after they count it. In reflection of my teaching, I need to take my time and model more of these principles when I am teaching my lesson, so students pick up on them, and then begin to use them regularly (Journal, Teacher 3KT, Dec. 13/13).

The 8 principals of counting were also interesting to me. I was not familiar with them. I did not realize that students may not realize the last number that we counted represented how many there were in total. Nor did I think that the space objects covered may influence students to think there were more or less of an item. We have begun to work on these principles during morning message, play based centres and math lessons, as well as whenever the opportunities presents itself throughout the day (Journal, Teacher 3LT, Dec. 13/13).

Educators became more aware of how they wanted to continue to use language in relation to how they talked about numbers. Their consolidated growth regarding math vocabulary and language, and its role in math instruction, is a meaningful improvement that they intend to also apply to future practices.

Release Date Nov. 28.... This session also made a change in how I talk about numbers. Instead of saying 23 "is a 2 then a 3" I will work at saying 23 "the 2 is 2 groups of 10 and 3". I also want to make a greater than, less than, and equal to magnets with popsicle sticks, and make them into monster. I noticed in our data that students did not understand greater then, or less then. I will use the <,>, = symbols during our circle times (Journal, Teacher 2AT, Jan. 18/14).

Now my thinking is this: so say we had 25. Normally I would just say, if we are writing it, I would say, "a 2 and a 5". So my language and my thinking is now I always ask, well "what does this number represent? If the number is 25, what does the 5 represent and what does the 2 represent?" A lot of my kids now know that 2 means there's two groups of 10 where before they thought it was just a 2 and a 5. So I think that's on a regular basis, they are more conscious of that and then even my wording to them too... it's more of a growth for me in the language that I use: fewer, more, less than. I probably wouldn't have thought of before, but it's more on my mind now (Interview, Early Childhood Educator 2AE).

Ok, for me the strategies and the discourse, they go together... the discourse in how we talk and the language we use, the vocabulary we use... [Facilitator] was telling us about numbers versus numerals so... I'm thinking maybe next year I'll start to do it with numerals... We also talked about less versus fewer... and it got me thinking, so when I talk to them, I watch how I speak and then I tell how they should be saying it, not that they have to (Interview, Teacher 4VT).

Another resolve among educators in relation to their use of evidence-based practices was to continue to teach number sense using inquiry and multiple strategies to accommodate for students' individual differences. In this way they are supporting student achievement by embodying a growth mindset as they continually find new ways to teach concepts and to expand their repertoire of interactive instructional approaches.

It's using different approaches with different kids, sitting down with them and getting to know them. In September once we got to know the difference from this child and that child, there were different ways of getting them to tell you what those numbers are. It's just using different approaches. Before, I always thought that when I sit down with them [students], we are going to do it *this way*. And then after sitting down and meeting with [the facilitator] and we took different approaches to try and get what we were looking for... Just trying different ways and not always doing it the same way (Interview, Early Childhood Educator, 2BE).

I would like some more ways to make games *or interactive actives* with manipulatives. I did get the book that has games in it and I have just now done some of them and but I didn't have enough time to do a lot of them... It's beneficial because that is how some of them learn, everybody learns differently. Some of them are hands-on learners, so it appeals to a lot of different learning styles (Interview, Early Childhood Educator 3LE).

> I did have students gain in areas such as place value. This was a focus for me since the initial testing as we did not score well. I will continue to work with my students in these areas and will continue to use inquiry in the classroom (Journal, Teacher 3MT, Feb. 25/14).

The new approach to embedding and integrating number sense into other strands was a compelling way to make instruction around number sense more meaningful and relevant to learners. Since this approach was a more efficient way to cover curriculum, and also provided an engaging route for teaching number sense in a context, it was clear that educators will not revert to old methods of teaching number sense concepts in isolation.

> Well if I think right now, before we used to look at all of the different strands and teach them separately. Now I find I am more open to relating them. Like number sense and numeration seems to be the base for everything, and you can kind of take and put it in everything else: into patterning, into graphing... (Interview, ELKP Teacher 4VT).

> At the beginning when we were here with [facilitator] he told us to open up the curriculum, go through the math and point out where math and number sense is in every strand. I'm thought... it's just in number sense. I've been teaching for a long time. But then I realized, "Oh my goodness how come I never saw this? Why didn't I pull this apart before? It's in geometry, it's in patterning, and it's everywhere". So, now it's not just number sense separately, now it's everywhere... As a teacher I don't have to just focus on just number sense alone, I focus on number sense everywhere (Interview, ELKP Teacher 3KT).

Students' Achievement

Were there gains in students' conceptual understanding and achievement as a function of the evidence-based practices?

Educators did see marked improvements in students' understanding and achievement of concepts related to number sense. Their expressed commitment to revise their teaching strategies around decomposing and composing numbers, and to generally ensure they have sufficiently covered the 8 principles of counting was a catalyst for improved student achievement.

> I would say that even then decomposing and composing numbers now, they are so much more familiar with it. We would do random things now where we'd have like in our story it would say the five ducks and I'd say "Oh! Show me on your fingers how can we see 5 ducks." I would have just done five and zero, but they know there are many ways now of making these numbers... Just because I think it's 5 and zero doesn't mean it always is. So kids know there's other ways to represent and understand... They also know that if they're wrong, they're not in trouble for being wrong. We can help them, you know we might say, "Who can help them? Who can offer some help?" They love being able to help their friends... they have become more confident over the year (Interview, Early Childhood Educator 2AE).

> The breaking down of the number, like the number 10. I would have never of thought that at that age they could deconstruct them... I know they have improved... So when they first got the buckets [manipulatives for their inquiry activities] some of them were like "what do I do?" If they had blocks they were building with them, they were making towers and things like that with no rhyme or reason to them. But after a little while they were sorting, and if they built something they built it red, blue, then red, blue, then red, blue type of thing. They had patterns within their structure... Also they can match and see that the five does not have to be 5 in a row, it could be 3 up and 2 down. It could be different ways, but it is still 5 (Interview, Early Childhood Educator 3LE).

> Overall I am pleased with their recognition of what a number is and how to break it down. So I saw a lot of gains that way, and in their confidence. As a whole, I saw an improvement in their confidence. I have one little guy, who was very unsure, very nervous, and constantly looking for reassurance asking me, "Is this what I do? Is this what I do?" And he's blossomed. He couldn't even find the page before if it was a high digit number, he didn't even know where to look. Now he is getting to the page, he's waiting for the instruction, but then he is doing it on his own. And he understands the number concepts (Interview, Grade 2 Teacher 2HT).

> I have noticed it more in our adding numbers together in word problems, that is where I have seen a big change and how its help them (Interview, Grade 1 Teacher 2FT).

Educators have also noticed that students can transfer skills to new settings and have been able to demonstrate skills on their own initiative without prompting. This achievement means that students have internalized new learning into their repertoire of knowledge, and verifies that student achievement has authentically extended beyond superficial levels characterized by rote learning or memorization.

> We noticed that they started doing patterning and things like that on their own now... And the other day someone came over and said to me, “look Mrs. [name], I’m making a pattern. Do you know what happens next? There is 2 blue and 2 brown. What happens after the brown? Do you know Mrs. [name]?” and I purposely said a wrong answer to see if they'd know. So they said, “Oh no that’s not the right colour Mrs. [name]. It’s 2 blue, 2 brown and *then 2 blue again"*. So I said, “oh ok, thank you for letting me know”. You can tell that they’ve learned it, they’ve grasped it and now they are roll-playing it in all their different centers so it’s really nice to see that... They play teacher a lot at the front of the classroom with our white board... they pretend they are teachers. We taught them, so now they are trying to teach all their little buddies. It’s kind of cute that they’ve grasped it and are now teaching it to all their friends (Interview, Early Childhood Educator 4QE).

> We grew different plants in our classroom and we would measure them and I had a girl the other day and she said “Look this one is so big now it’s longer than the ruler”. I said, "how can we see then how much more it is?" It’s cool just on her own that she noticed these things (Interview, Early Childhood Educator 2AE).

> I have really started to notice that the students start to naturally make patterns, sort, measure and count. They enjoy the play time, and see my prompts and challenges as fun instead of feeling like it is work. They have begun to have conversations about Math with peers and now, when they are unable to solve a problem, they are more open to talking to a peer next to them to work it out... We need to give time to explore Math during play, but they also need time to reflect on it and then discuss it with their peers (Journal, Teacher 3LT, Feb. 25/14).

Students appear to be more confident in math, more comfortable with open ended tasks, and are willing to take risks on their own by engaging in various activities. Undoubtedly, educators' renewed focus on process, rather than product, and their stance that mistakes are a productive part of a successful learning experience has cultivated a positive learning environment where individual students' processes are valued and encouraged, which has fostered a risk-free environment promoting student engagement and willing participation in activities.

> I think the gains for my students... one thing I noticed before is 'math is math' and a lot would think 'I am good at it or not'. One thing that I didn’t see now is students not wanting to do math. But whether it be because it was not pencil and paper, they really seem to enjoy it... In general, they feel more confident, maybe, but they are definitely willing to jump into math (Interview, Grade 1/2 Teacher 3NT).

> We’ve seen a lot of comfort in open ended tasks... Like even just building, there is a few that love to go with cubes. They always go to cubes, so now we always ask them either make a plan, or try to relate it to something else that we were working on. We were learning about insects, and one little boy, no one told him anything, he went right to the cubes and built a dragon fly. He made sure he used a pattern, I watched him. He counted to make sure the wings were exactly the same length. He used same amount of cubes, he used different ones for the eyes and he made sure they stuck out. He just had a lot of skills, a lot of concepts being used (Interview, ELKP Teacher 4VT).

Quantitative results complement these qualitative findings. The results of the quantitative analyses computed using the students’ assessment data are displayed in Tables 3. to 6. The tables provide a summary of both the significant and non-significant findings of the Wilcoxon Signed-Rank Test for each of the question items on the *Developmental Math Assessment*. Results are displayed

when scores for all schools (at each of the grades) are combined together. There is an interpretation of the significant results in the final column.

DMA Sub-tests	Wilcoxon Statistics	Overall Interpretation
Oral Counting	Z = -3.15, p = 0.00 Median=2.0	There was a statistically significant change in **Oral Counting** sub-test scores from the beginning of the school year to the end of the school year.
Object Counting (1-1)	Z = -2.12, p = 0.03 Median=2.0	There was a statistically significant change in **Object Counting (1-1)** sub-test scores from the beginning of the school year to the end of the school year.
Object Counting (cardinality)	Z = -3.31, p = 0.00 Median=2.0	There was a statistically significant change in **Object Counting (cardinality)** sub-test scores from the beginning of the school year to the end of the school year.
Visual Patterns	Z = -3.31, p = 0.00 Median=2.0	There was a statistically significant change in **Visual Patterns** sub-test scores from the beginning of the school year to the end of the school year.
Compare (more in a group)	Z = -2.45, p = 0.01 Median=2.0	There was a statistically significant change in **Compare (more in a group)** sub-test scores from the beginning of the school year to the end of the school year.
Matching	Z = -0.71, p = 0.48 Median=2.0	There was NOT a statistically significant change in **Matching** sub-test scores from the beginning of the school year to the end of the school year.

Table 4. Comparison of Pre-Test to Post-Test ELKP (Year 1) All Students at All Schools

There was a significant growth in student performance for all ELKP (Year 1) students at all three schools in the following five concepts: Oral Counting; Object Counting (1-1); Object Counting (cardinality);Visual Patterns; Comparing (more in a group). There was NOT growth in concept of Matching.

DMA Sub-tests	Wilcoxon Statistics	Interpretation
Oral Counting	Z = -1.41, p = 0.16 Median=2.0	There was NOT a statistically significant change in **Oral Counting** sub-test scores from the beginning of the school year to the end of the school year.
Object Counting (1-1)	Z = -2.33, p = 0.02 Median=2.0	There was a statistically significant change in **Object Counting (1-1)** sub-test scores from the beginning of the school year to the end of the school year.
Object Counting (cardinality)	Z = -1.67, p = 0.09 Median=2.0	There was NOT a statistically significant change in **Object Counting (cardinality)** sub-test scores from the beginning of the school year to the end of the school year.
Object Counting (using a plan)	Z =-2.00, p = 0.05 Median=2.0	There was a statistically significant change in **Object Counting (using a plan)** sub-test scores from the beginning of the school year to the end of the school year.
Identify Symbols	Z =-4.24, p = 0.00 Median=2.0	There was a statistically significant change in **Identifying Symbols** sub-test scores from the beginning of the school year to the end of the school year.

One More (come after)	Z=-1.15, p = 0.25 Median=2.0	There was NOT a statistically significant change in **One More (come after)** sub-test scores from the beginning of the school year to the end of the school year.
One More (one more)	Z=-2.60, p = 0.01 Median=2.0	There was a statistically significant change in **One More (one more)** sub-test scores from the beginning of the school year to the end of the school year.
One Less	Z=-1.00, p = 0.32 Median=2.0	There was NOT a statistically significant change in **One Less** sub-test scores from the beginning of the school year to the end of the school year.
Visual Patterns	Z= -4.24, p = 0.00 Median=2.0	There was a statistically significant change in **Visual Patterns** sub-test scores from the beginning of the school year to the end of the school year.
Compare (more in a group)	Z= -1.89, p = 0.06 Median=2.0	There was NOT a statistically significant change in **Compare (more in a group)** sub-test scores from the beginning of the school year to the end of the school year.
Compare (greater number)	Z=-0.63, p = 0.53 Median=2.0	There was NOT a statistically significant change in **Compare (greater number)** sub-test scores from the beginning of the school year to the end of the school year.
Sort	Z=-3.05, p = 0.00 Median=2.0	There was a statistically significant change in **Sort** sub-test scores from the beginning of the school year to the end of the school year.
Matching	Z= -4.95, p = 0.00 Median=0 at pre-test Median=2.0 at post-test	There was a statistically significant change in **Matching** sub-test scores from the beginning of the school year to the end of the school year.
Positional Words	Z=-2.00, p = 0.05 Median=2.0	There was a statistically significant change in **Positional Words** sub-test scores from the beginning of the school year to the end of the school year.

Table 5. Comparison of Pre-Test to Post-Test ELKP (Year 2) All Students at All Schools

There was a significant growth in student performance for all ELKP (Year 2) students at all three schools in the following eight concepts: Object Counting (1-1); Object Counting (using a plan); Identify Symbols; One More; Visual Patterns; Sorting; Matching; Positional Words. There was NOT growth in concepts of Oral Counting; Object Counting (cardinality); One More (come after); One Less; Compare (more); Compare (greater).

DMA Sub-tests	Wilcoxon Statistics	Interpretation
Oral Counting	Z=-0.50 , p = 0.62 Median=2.0	There was NOT a statistically significant change in **Oral Counting** sub-test scores from the beginning of the school year to the end of the school year.
Oral Counting (next decade)	Z=-2.83, p = 0.01 Median=0 at pre-test Median=2.0 at post-test	There was a statistically significant change in **Oral Counting (next decade)** sub-test scores from the beginning of the school year to the end of the school year.
Oral Counting (by 10's)	Z=-3.61, p = 0.00 Median=2.0	There was a statistically significant change in **Oral Counting (by 10's)** sub-test scores from the beginning of the school year to the end of the school year.

Oral Counting (come after: next decade)	Z=--2.31, p = 0.02 Median=2.0	There was a statistically significant change in **Oral Counting (come after)** sub-test scores from the beginning of the school year to the end of the school year.
Object Counting (1-1)	Z=-1.13, p = 0.26 Median=2.0	There was NOT a statistically significant change in **Object Counting (1-1)** sub-test scores from the beginning of the school year to the end of the school year.
Object Counting (cardinality)	Z=- 1.63, p = 0.10 Median=2.0	There was NOT a statistically significant change in **Object Counting (cardinality)** sub-test scores from the beginning of the school year to the end of the school year.
Object Counting (using a plan)	Z=-2.50, p = 0.01 Median=2.0	There was a statistically significant change in **Object Counting (using a plan)** sub-test scores from the beginning of the school year to the end of the school year.
Identify Symbols	Z=-3.42, p = 0.00 Median=2.0	There was a statistically significant change in **Identifying Symbols** sub-test scores from the beginning of the school year to the end of the school year.
Write Symbols	Z=-2.60, p = 0.01 Median=2.0	There was a statistically significant change in **Writing Symbols** sub-test scores from the beginning of the school year to the end of the school year.
One More	Z=-2.19, p = 0.03 Median=2.0	There was a statistically significant change in **One More** sub-test scores from the beginning of the school year to the end of the school year.
One Less (comes before)	Z=-1.51, p = 0.13 Median=2.0	There was NOT a statistically significant change in **One Less (comes before)** sub-test scores from the beginning of the school year to the end of the school year.
One Less (one less)	Z=-3.05, p = 0.00 Median=0 at pre-test Median=2.0 at post-test	There was a statistically significant change in **One Less (one less)** sub-test scores from the beginning of the school year to the end of the school year.
Visual Patterns	Z=-3.28, p = 0.00 Median=2.0	There was a statistically significant change in **Visual Patterns** sub-test scores from the beginning of the school year to the end of the school year.
Compare (more in a group)	Z=-0.69, p = 0.49 Median=2.0	There was NOT a statistically significant change in **Compare (more in a group)** sub-test scores from the beginning of the school year to the end of the school year.
Compare (greater number)	Z=-0.45, p = 0.66 Median=2.0	There was NOT a statistically significant change in **Compare (greater number)** sub-test scores from the beginning of the school year to the end of the school year.
Order Numbers	Z=-2.52 , p = 0.01 Median=0 at pre-test Median=2.0 at post-test	There was a statistically significant change in **Order Numbers** sub-test scores from the beginning of the school year to the end of the school year.
Compose (how many to 10)	Z= -3.13, p = 0.00 Median=0 at pre-test Median=2.0 at post-test	There was a statistically significant change in **Compose (how many to 10)** sub-test scores from the beginning of the school year to the end of the school year.

Decompose (ways to make 10)	Z=-4.49, p = 0.00 Median=0 at pre-test Median=2.0 at post-test	There was a statistically significant change in **Decompose (ways to make 10)** sub-test scores from the beginning of the school year to the end of the school year.
Place and Value	Z=-2.65, p = 0.01 Median=0	There was a statistically significant change in **Place and Value** sub-test scores from the beginning of the school year to the end of the school year.

Table 6. Comparison of Pre-Test to Post-Test Grade 1 All Students at All Schools

There was a significant growth in student performance for all Grade 1 students at all three schools in the following thirteen concepts: Oral Counting (next decade); Oral Counting (by 10's); Oral Counting (come after); Object Counting (use a plan); Identify Symbols; Write Symbols; One More; One Less (less); Visual Patterns; Order Numbers; Compose; Decompose; Place and Value. There was NOT growth in concepts of Oral Counting; Object Counting (1-1); Object Counting (cardinality); One Less; Compare (more); Compare (greater).

DMA Sub-tests	Wilcoxon Statistics	Interpretation
Oral Counting	Z=-0.82, p = 0.41 Median=2.0	There was NOT a statistically significant change in **Oral Counting** sub-test scores from the beginning of the school year to the end of the school year.
Oral Counting (next decade)	Z=-1.63, p = 0.10 Median=2.0	There was NOT a statistically significant change in **Oral Counting (next decade)** sub-test scores from the beginning of the school year to the end of the school year.
Object Counting (1-1)	Z=-1.41, p = 0.16 Median=2.0	There was NOT a statistically significant change in **Object Counting (1-1)** sub-test scores from the beginning of the school year to the end of the school year.
Object Counting (counts by 10's)	Z=-2.83, p = 0.01 Median=2.0	There was a statistically significant change in **Object Counting (counts by 10's)** sub-test scores from the beginning of the school year to the end of the school year.
Object Counting (unitizing)	Z=-1.94, p = 0.05 Median=2.0	There was a statistically significant change in **Object Counting (unitizing)** sub-test scores from the beginning of the school year to the end of the school year.
Identify Symbols	Z=-3.67, p = 0.00 Median=1 at pre-test Median=2.0 at post-test	There was a statistically significant change in **Identifying Symbols** sub-test scores from the beginning of the school year to the end of the school year.
Write Symbols	Z=-3.15, p = 0.00 Median=1 at pre-test Median=2.0 at post-test	There was a statistically significant change in **Writing Symbols** sub-test scores from the beginning of the school year to the end of the school year.
One More	Z=-1.51, p = 0.13 Median=2.0	There was NOT a statistically significant change in **One More** sub-test scores from the beginning of the school year to the end of the school year.
Ten More	Z=-2.83, p = 0.01 Median=0	There was a statistically significant change in **Ten More** sub-test scores from the beginning of the school year to the end of the school year.

One Less	Z=-3.38, p = 0.00 Median=0 at pre-test Median=2.0 at post-test	There was a statistically significant change in **One Less** sub-test scores from the beginning of the school year to the end of the school year.
Ten Less	Z=-2.67, p = 0.01 Median=0	There was a statistically significant change in **Ten Less** sub-test scores from the beginning of the school year to the end of the school year.
Comes Before	Z=-3.00, p = 0.00 Median=0 at pre-test Median=2.0 at post-test	There was a statistically significant change in **Comes Before** sub-test scores from the beginning of the school year to the end of the school year.
Visual Patterns	Z=-2.53, p = 0.01 Median=1 at pre-test Median=2.0 at post-test	There was a statistically significant change in **Visual Patterns** sub-test scores from the beginning of the school year to the end of the school year.
Compare Numbers	Z=-2.74, p = 0.01 Median=2.0	There was a statistically significant change in **Compare Numbers** sub-test scores from the beginning of the school year to the end of the school year.
Order Numbers	Z=0.00, p = 1.00 Median=2.0	There was NOT a statistically significant change in **Order Numbers** sub-test scores from the beginning of the school year to the end of the school year.
Compose/Decompose	Z=-3.31, p = 0.00 Median=0 at pre-test Median=1.5 at post-test	There was a statistically significant change in **Compose/Decompose** sub-test scores from the beginning of the school year to the end of the school year.
Place and Value (making number)	Z=-1.21, p = 0.23 Median=2.0	There was NOT a statistically significant change in **Place and Value (making number)** sub-test scores from the beginning of the school year to the end of the school year.
Place and Value (meaning of digit)	Z=-2.89, p = 0.00 Median=0	There was a statistically significant change in **Place and Value (meaning of digit)** sub-test scores from the beginning of the school year to the end of the school year.

Table 7. Comparison of Pre-Test to Post-Test Grade 2 All Students at All Schools

There was a significant growth in student performance for all Grade 2 students at all three schools in the following twelve concepts: Object Counting (counts by 10's); Object Counting (unitizing); Identify Symbols; Write Symbols; Ten More; One Less (less); Ten Less; Comes Before; Visual Patterns; Compare Numbers; Compose/Decompose; Place and Value (meaning of digit). There was NOT growth in concepts of Oral Counting; Oral Counting (next decade); Object Counting (1-1); One More; Order Numbers; Place and Value (making number).

Limitations

Educational research is inherently open to limitations due to the nature of the work with human participants. The study examined here exhibits some limitations that merit noting. There is the risk of response bias of social desirability when the teachers, early childhood educators, and facilitator were interviewed (Creswell, 2012). Specifically, the interviewees might have filtered their responses in a conscious effort to create a favourable impression of participating in this project. There is an even stronger tendency for participants to modify their responses when they are not anonymous (Creswell, 2012). A related concern might be from the self-report measure that the teachers completed. Likert-scale surveys such as the one used to capture teachers' beliefs about mathematics instruction are

susceptible to biased self-reporting (Creswell, 2012). Another limitation might stem from the researchers (and facilitator) being present during the observation, co-planning, and meetings. The Hawthorne effect (i.e., participants may change their typical behaviour merely because of the presence of the researchers) may have limited results (Creswell, 2012).

The student assessment data presented some minor limitations to the analyses as in some cases it was incomplete due to student mobility, attrition or lack of final parental consent. As well, this research also included a small number of teacher participants from a medium-sized school board which results in a low level of generalizability. There was also no random assignment of teachers to a control or intervention condition. This research could thus be enhanced using a quasi-experimental design with the inclusion of a teacher (non-intervention) control group. Finally, there is a lack of data from the perspective of the student learners in situ. Teachers commented on students` attitudes and behaviours; it would be optimal to garner the students` voices and examine their learning in the classroom.

Next Steps

Lessons Learned: Implications for Practice

After a facilitated learning session of curriculum mapping, educators realized the extent to which curriculum expectations in number sense are interrelated with other curriculum expectations in math, and in other subjects. This fueled their ability to create authentic learning opportunities for their students in which number sense concepts were taught in a relevant context, and not simply as an isolated skill. The contextualized learning fostered students' understanding beyond rote learning mechanics, and propelled their meaningful understanding of number sense to a deeper level. Consequently, students were able to generalize their learning to the extent that they demonstrated transfer of skills to a variety of other contexts. Since the catalyst to this rich learning environment was a curriculum mapping exercise, in order to promote educators' integrated instructional practices, facilitators should ensure that educators are explicitly aware of curricular connections. An unexpected positive outcome of integrated instruction was that it alleviated some stress pertaining to covering curriculum within concrete time constraints, as educators saved time by layering concepts taught concurrently. This positive implication for practice should be made apparent to educators at the beginning of the project to assuage any potential concerns about covering the breadth of concepts. Professional learning facilitators might lead curriculum mapping in-services or provide documents detailing curriculum connections to educators.

The success of inquiry-based approaches was contingent on educators feeling comfortable adjusting their traditional classroom role from a teacher-driven environment to one that incorporated student-driven practices. Although it may have been a difficult adjustment at times, the benefits of allowing students to take ownership over their own learning proved to be worthwhile. Educators observed students' excitement during student-led activities, which ultimately translated in more engaged learners and improved student achievement. When introducing inquiry-based processes, facilitators should overtly discuss the critical transition of educators' orientation from teacher-driven to student-driven practices so that a forum for a discussion of best practices around the classroom conditions needed for successful inquiry-based practices can ensue. At the beginning of a program of professional learning, facilitators should avoid the pragmatics of specific inquiry-based strategies and begin with a discussion of how to foster a student-driven classroom culture that supports such inquiry-based strategies.

One of the most effective ways to support educators' adoption of new practices in their classroom is to have the innovative strategies modeled for them in an authentic classroom setting. While many educators were open to trying the new strategies or approaches discussed at facilitated learning sessions, many lacked either the confidence or the instructional skill set to implement the innovative strategies until they directly observed a classroom demonstration of such strategies. Further, the fruitful collegial discussions around best practices that classroom demonstrations launch were cited as instrumental to educators' gravitation towards the implementation of new strategies. Therefore, in

order for educators to perceive the implementation of innovative practices as viable in their own classroom, facilitators should couple their discussions pertaining to instructional strategies with opportunities for educators to directly observe them being modeled in the classroom. Professional learning facilitators should prioritize on-site instructional modeling and demonstrations of new strategies to set the stage for educators to translate into their own practice.

An expansion of the current program of professional learning might include a blend of teachers that have previously participated and teachers new to the initiative. In a family of geographical related schools, teachers might co-plan, model, and co-teach early learning math lessons. An on-line forum for sharing resources, recommendations and feedback would support this professional learning community. Facilitators such as the Numeracy Facilitator, Numeracy Consultant and Math Coaches could offer on-going support to the educators. Both teachers and students who participate in the 2013-2014 school year should be monitored with data collection (e.g., interviews, student math assessments).

A comprehensive assessment of students' knowledge, along with a clear understanding of developmental stages and learning trajectories, enabled educators to strategically determine a data-driven plan for individual student achievement. The detailed inventory of an individual student's knowledge base early in the school year was instrumental information for the development of student success plans. While the challenge of completing rigorous diagnostic assessments for each student is time consuming, since this baseline data is valuable information for instructional planning purposes, it is a task that needs to be prioritized. Therefore, if true meaningful, data-driven instructional decisions are to be made, comprehensive baseline data about students' knowledge in relation to curricular expectations needs to be collected early in the school year.

The findings of this project evaluation have provided quantitative evidence that an instructional focus on specific concepts in number sense and numeration enhance student performance on a criterion-based assessment. Such integrity between assessed and identified concepts and instruction is the key principle underlying curricular redesign and instructional planning. This premise should be at the forefront in preparing for subsequent iterations of this project. The investment in solidifying the assessment to instruction cycle will reap dividends as the early learning challenges and uncorrected misconceptions in mathematics will be identified and addressed. This will support students' early learning success in mathematics, encourage risk-taking during problem solving, and inoculate against poor math self-efficacy.

Implications for Future Research

The paucity of longitudinal research tracking the effects of teacher professional learning on both educators' practice and students' mathematics achievement means that little is yet known about the potential for establishing enduringly effective professional learning communities. An opportunity exists for a sustained investigation into the program of mathematics professional learning that has been presented herein. The teachers that have served as participants for this portion of the project are well-positioned to further enhance their mathematics instruction and be tracked into the next academic year. This could adopt a quasi-experimental research design with a non-intervention group or an intimate case study design. This also poses interesting prospects as some teachers enter their first, second or third year of the project. The achievement of their former and present students could also be studied. The research design could be extended to include additional teachers who might be mentored by the experienced ones. This type of parallel professional learning is collegial and affirming for all participants and would make a significant contribution to the research literature.

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Dr. Marybeth Fortune is an educational researcher and instructor in the Department of Teacher Education at Brock University. Her research expertise includes the investigation of leadership strategies in relation to the implementation of improvement initiatives and the management of organizational change.

A request was made by NCDSB to the Research Officer at Brock University Faculty of Education for researchers to investigate the *Early Learning Math Inquiry Project* and together Drs. Gallagher and Fortune elected to assume this task. Throughout the 2013/2014 school year, Drs. Gallagher and Fortune have remained at arms-length to the design and facilitation of this project. The research of this project was vetted through the Research Ethics Board at Brock University and NCDSB. The researchers independently and confidentially collected and analyzed all of the data contained in this report. Drs. Gallagher and Fortune have never been employed by Niagara Catholic District School Board and were not remunerated for the research or writing of this report; consequently they have remained objective evaluators throughout this process.